# Work, energy and power: study notes

IB Physics HL · A.3

**This topic is the accounting system of mechanics: work measures how much energy a force transfers, energy is stored in a few named forms, and power is how fast the transfer happens. In Paper 1 you will meet one-step items on work from a graph, power at constant speed and efficiency; in Paper 2 an object is followed along a track or through the air and you are asked for a speed by energy conservation, the work done against friction as a difference of mechanical energies, or the power of a motor lifting at constant speed. The calculations are short; the marks are won by choosing the right energy relation, the right reference level and consistent units.**

## 1. Work done by a constant force

Work is the energy transferred by a force. For a constant force $F$ whose line of action makes an angle $\theta$ with the displacement $s$,

$$W = Fs\cos\theta$$

(in the booklet). Work is a scalar measured in joules ($1\ \text{J} = 1\ \text{N m}$). Only the component of the force along the displacement does work. Three cases to recognise instantly: $\theta = 0°$ gives $W = Fs$ (force pulls along the motion, positive work); $\theta = 90°$ gives $W = 0$ (a normal reaction on a fixed surface, the tension in a string swinging a bob in a circle, or any force perpendicular to the motion); $\theta = 180°$ gives $W = -Fs$ (friction or air resistance opposing the motion). Sign convention (learn it, not in the booklet): work done by a force is positive when the force has a component along the displacement; when we say work is done against a force such as friction, we quote the magnitude $Fs$ and describe it as energy removed from the mechanical store. State which force you are calculating work for and use the angle between the force and the displacement, not the angle a slope makes with the ground unless that is the same thing.

## 2. Work from a force-displacement graph and variable forces

When the force changes with position, $W = Fs\cos\theta$ cannot be used directly. Instead the work done equals the area between the force-displacement graph and the displacement axis (learn it, not in the booklet). For a rectangle (constant force) the area is $F \times s$; for a triangle (force rising linearly from zero, as for a spring) the area is $\frac{1}{2}F_{\text{max}}\Delta x$; for a curve, count squares or split the shape into trapezia. Areas below the axis count as negative work. The spring is the standard example: the force needed to stretch an ideal spring is $F = k\Delta x$, so the graph is a straight line through the origin with gradient $k$, and the area up to extension $\Delta x$ is $\frac{1}{2}k\Delta x^2$, which is exactly the elastic potential energy stored. In a Paper 1 item the trap is to read the force at the end of the motion and multiply by the whole distance; always ask whether the force was constant over the interval.

## 3. The energy principle: forms, transfers and conservation

Energy is stored in named forms: kinetic (a moving mass), gravitational potential (a mass raised in a field), elastic potential (a deformed spring or material), thermal (random motion of particles, raised temperature), chemical (fuel, food, batteries), nuclear (binding of nucleons) and electrical (charges moved through a potential difference). A transfer moves energy between objects; a transformation changes its form. The principle of conservation of energy (learn the statement) says that in a closed system the total energy is constant: energy is neither created nor destroyed, only transferred or transformed. The work-energy theorem (learn it) is the mechanical version: the net work done on a body

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by all forces equals its change in kinetic energy,

$$W_{\text{net}} = \Delta E_k = \frac{1}{2}mv^2 - \frac{1}{2}mu^2.$$

Two equivalent ways of attacking a problem follow. Either add up the work done by every force (gravity, tension, friction, normal reaction) and set the total equal to $\Delta E_k$, or write the mechanical energy $E_k + E_p$ at two points and let the difference equal the work done against resistive forces. Both routes give the same answer; the second is faster for tracks and projectiles because gravity is handled through $E_p$ without resolving forces along the path.

## 4. Kinetic, gravitational potential and elastic potential energy

Kinetic energy:

$$E_k = \frac{1}{2}mv^2 = \frac{p^2}{2m}$$

(in the booklet). The momentum form is useful when two particles share the same momentum, for example after an explosion from rest: the lighter one takes more kinetic energy because $E_k \propto 1/m$ at fixed $p$. Note that doubling the speed quadruples the kinetic energy, a favourite Paper 1 ratio. Gravitational potential energy near the surface:

$$\Delta E_p = mg\Delta h$$

(in the booklet), valid where $g$ is uniform. Only changes in height matter, so choose a reference level (usually the lowest point) and measure every $h$ from it. The force of gravity does work $mg\Delta h$ on a falling body, which is why a free-fall speed comes from $\frac{1}{2}mv^2 = mg\Delta h$, independent of mass. Elastic potential energy stored in a spring of constant $k$ at extension or compression $\Delta x$:

$$E_H = \frac{1}{2}k\Delta x^2$$

(in the booklet). The symbol $E_H$ is the booklet symbol for elastic energy. When a spring launches a trolley, $\frac{1}{2}k\Delta x^2$ becomes $\frac{1}{2}mv^2$ (plus any work against friction); when a block drops onto a vertical spring, gravitational energy becomes elastic energy at maximum compression, and the height fallen includes the compression itself.

## 5. Power

Power is the rate of doing work or transferring energy:

$$P = \frac{\Delta W}{\Delta t}$$

(in the booklet), measured in watts ($1\ \text{W} = 1\ \text{J s}^{-1}$). For a constant force acting on a body moving at speed $v$ along the force,

$$P = Fv$$

(in the booklet), which follows from $W = Fs$ divided by $t$. Two exam settings dominate. A motor or crane lifting a load at constant speed: the lifting force equals the weight (zero acceleration), so $P = mgv$. A car or cyclist at constant speed on a level road: the driving force equals the total resistive force, so $P = F_{\text{resist}}v$; if the speed doubles and the whole of the resistance is air resistance proportional to $v^2$, the power needed rises by a factor of eight. Always state what a power value refers to (the useful power to the load, or the power supplied to the motor) and the speed or time interval it belongs to. Watch prefixes: 3.5 kW = 3500 W.

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## 6. Efficiency, friction and Sankey diagrams

Efficiency is the fraction of the input that appears as useful output:

$$\text{efficiency} = \frac{\text{useful work out}}{\text{total work in}} = \frac{\text{useful power out}}{\text{total power in}}$$

(in the booklet). It has no unit and is quoted as a decimal or a percentage; it can never exceed 1 because energy is conserved. Friction and air resistance transform mechanical energy (kinetic plus potential) into thermal energy of the surfaces and the air. Energy is still conserved, but the thermal energy is spread out and cannot be recovered to do useful work, which is why we call it wasted. The work done against friction over a stretch of track is the loss of mechanical energy:

$$W_{\text{friction}} = (E_k + E_p)_{\text{start}} - (E_k + E_p)_{\text{end}},$$

and dividing by the distance travelled gives the average frictional force. A Sankey diagram (learn how to draw and read one) shows energy or power flow as arrows whose widths are proportional to the quantity: a single input arrow on the left, the useful output continuing straight to the right, and each wasted form branching off, usually downwards. The widths must add up: the input width equals the sum of all output widths. To read efficiency from a Sankey diagram, divide the width of the useful arrow by the width of the input arrow.

### Formulas

| | |
|---|---|
| $W = Fs\cos\theta$ | in the booklet |
| $E_k = \frac{1}{2}mv^2 = \frac{p^2}{2m}$ | in the booklet |
| $\Delta E_p = mg\Delta h$ | in the booklet |
| $E_H = \frac{1}{2}k\Delta x^2$ | in the booklet |
| $P = \frac{\Delta W}{\Delta t} = Fv$ | in the booklet |
| $\text{efficiency} = \frac{\text{useful work out}}{\text{total work in}} = \frac{\text{useful power out}}{\text{total power in}}$ | in the booklet |
| $W_{\text{net}} = \Delta E_k$ (work-energy theorem) | learn it |
| Work = area under the force-displacement graph | learn it |
| $W_{\text{friction}} = (E_k + E_p)_{\text{start}} - (E_k + E_p)_{\text{end}}$ | learn it |
| Total energy of a closed system is constant | learn it |

### Worked examples

**Example 1.** A crate of mass 12 kg is pulled from rest across a horizontal floor by a rope. The tension in the rope is 60 N and the rope makes an angle of 30 degrees with the floor. The crate moves 8.0 m and a constant frictional force of 25 N acts on it. (a) Calculate the work done on the crate by the tension. (b) Calculate the work done on the crate by friction. (c) State the work done by the normal reaction and by the weight. (d) Determine the speed of the crate after 8.0 m.

1. (a) Use $W = Fs\cos\theta$ with the angle between the tension and the displacement: $W_T = 60 \times 8.0 \times \cos 30^\circ = 480 \times 0.866 = 416$ J.
2. (b) Friction acts at $180^\circ$ to the displacement: $W_f = 25 \times 8.0 \times \cos 180^\circ = -200$ J.
3. (c) The normal reaction and the weight are both perpendicular to the horizontal displacement, so each does zero work.
4. (d) Work-energy theorem: $W_{\text{net}} = 416 - 200 = 216\text{ J} = \frac{1}{2}mv^2$.

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5. $v^2 = \dfrac{2 \times 216}{12} = 36$, so $v = 6.0$ m s$^{-1}$.

**Answer:** (a) 416 J, (b) minus 200 J, (c) zero for both, (d) 6.0 m s$^{-1}$.

**Example 2.** A sledge of mass 70 kg is released from rest at the top of a slope 15 m above its base. The slope is 40 m long. (a) Show that, if friction is ignored, the speed of the sledge at the bottom is about 17 m s$^{-1}$. (b) The actual speed at the bottom is measured to be 12 m s$^{-1}$. Calculate the work done against friction on the slope. (c) Determine the average frictional force acting on the sledge.

1. (a) Take the base as the reference level. Mechanical energy at top equals mechanical energy at bottom: $mg\Delta h = \frac{1}{2}mv^2$, so $v = \sqrt{2g\Delta h}$.
2. $v = \sqrt{2 \times 9.81 \times 15} = \sqrt{294} = 17.2$ m s$^{-1}$, which is about 17 m s$^{-1}$.
3. (b) Initial mechanical energy: $E_p = 70 \times 9.81 \times 15 = 1.03 \times 10^4$ J. Final mechanical energy: $E_k = \frac{1}{2} \times 70 \times 12^2 = 5040$ J.
4. Work done against friction is the loss of mechanical energy: $W_f = 10300 - 5040 = 5.3 \times 10^3$ J.
5. (c) $W_f = F_f \times s$ along the slope, so $F_f = \dfrac{5260}{40} = 132$ N $\approx 130$ N.

**Answer:** (a) 17.2 m s$^{-1}$, (b) about 5.3 kJ transferred to thermal energy, (c) about 130 N.

**Example 3.** An electric motor on a crane raises a load of mass 450 kg vertically at a constant speed of 0.80 m s$^{-1}$. The electrical power supplied to the motor is 5.2 kW. (a) Calculate the useful power delivered to the load. (b) Determine the efficiency of the motor. (c) Calculate the gain in gravitational potential energy of the load in 10 s. (d) Outline the Sankey diagram for the motor.

1. (a) At constant speed the lifting force equals the weight: $F = mg = 450 \times 9.81 = 4.41 \times 10^3$ N.
2. $P = Fv = 4414.5 \times 0.80 = 3.53 \times 10^3$ W $= 3.5$ kW.
3. (b) Convert to the same unit before dividing: efficiency $= \dfrac{3530}{5200} = 0.68$, that is 68 %.
4. (c) In 10 s the load rises $0.80 \times 10 = 8.0$ m, so $\Delta E_p = 450 \times 9.81 \times 8.0 = 3.5 \times 10^4$ J; this equals $3530 \times 10$, as it must.
5. (d) One input arrow of width representing 5.2 kW enters from the left; a straight arrow of width 3.5 kW continues to the right labelled useful power to the load; an arrow of width $5.2 - 3.5 = 1.7$ kW branches downwards labelled power wasted as thermal energy in the motor and surroundings. Widths are drawn to scale so that input equals total output.

**Answer:** (a) 3.5 kW, (b) 0.68 (68 %), (c) 35 kJ, (d) input 5.2 kW splitting into 3.5 kW useful and 1.7 kW wasted, widths proportional.

## Common errors

- Using $W = Fs$ when the force is not along the displacement. Always identify the angle between the force and the direction of motion and include $\cos\theta$; for a rope at an angle this reduces the work done.
- Including the normal reaction in the work bookkeeping. It acts at right angles to the motion on any fixed surface, flat or curved, so it does no work; the same applies to the tension in a pendulum string.

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- Reading a variable force off the graph at one point and multiplying by the distance. Work done by a variable force is the area under the force-displacement graph; for a spring that is a triangle, not a rectangle.
- Quoting a power with no indication of which speed or time it refers to, or dividing a kilowatt input by a watt output. Convert everything to watts or everything to kilowatts before using the efficiency relation, and check the answer is less than 1.
- Measuring height from the wrong place. $\Delta E_p = mg\Delta h$ needs the height fallen or risen, not the total height of the track; fix a reference level at the start and use it throughout.
- Forgetting the initial kinetic energy. If the object is not released from rest, the energy conservation statement must contain $\frac{1}{2}mu^2$ on the starting side as well.

**Before the exam, can you...**

- Before the exam, can you write $W = Fs\cos\theta$ and state, with reasons, which forces do positive, negative and zero work on a block pulled across a floor?
- Before the exam, can you find the work done by a variable force from the area under a force-displacement graph, including a triangular spring graph that gives $\frac{1}{2}k\Delta x^2$?
- Before the exam, can you set $E_k + E_p$ at one point equal to $E_k + E_p$ at another point to find a speed, then use the difference of mechanical energies to find the work done against friction and the average frictional force?
- Before the exam, can you calculate the power of a motor lifting a load at constant speed with $P = mgv$ and convert correctly between W and kW?
- Before the exam, can you compute efficiency from useful and total power, draw a Sankey diagram with widths that add up, and explain why energy transferred to thermal stores is wasted even though total energy is conserved?

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