Simple harmonic motion: practice worksheet

IB Physics HL · C.1 · 50 minutes · 42 marks

Question 1

An oscillator performs simple harmonic motion with amplitude x_0, angular frequency ω and period T. At $t = 0$, its displacement is $+x_0$ and its velocity is zero. Select one option in each part.

(a) Write down the equation relating its acceleration a to its displacement x from equilibrium. A: $a = -\omega^2 x$; B: $a = +\omega^2 x$; C: $a = -\omega x$; D: $a = -\omega^2 x_0$. [1]

(b) Determine when the oscillator first passes through equilibrium and the direction of its velocity then. A: $T/4$, positive; B: $T/4$, negative; C: $T/2$, negative; D: $T/2$, zero. [1]

(c) Determine the fraction of the total oscillation energy that is potential energy when $x = x_0/2$. A: $1/2$; B: $3/4$; C: $1/4$; D: $\sqrt{3}/2$. [1]

Question 2

A laboratory demonstration contains a simple pendulum of length 0.640 m and a separate horizontal spring oscillator with mass 0.160 kg and spring constant 25.0 N m^{-1}. The pendulum oscillates through small angles and friction is negligible. Take $g = 9.81$ m s^{-2}.

(a) Calculate the period of the pendulum. [2]

(b) Calculate the period of the spring oscillator. [2]

Question 3

A moving marker has acceleration $a = -(25.0\ \text{s}^{-2})x$, where x is its displacement from equilibrium. Its amplitude is 0.0800 m. At $t = 0$, it passes through equilibrium with positive velocity.

(a) Determine the angular frequency and the period of the motion. [2]

(b) Write down an equation for x as a function of t, including the phase angle. [1]

(c) Determine the first time after $t = 0$ at which $x = +0.0400$ m. [2]

(d) State the phase relationship between velocity and displacement. [1]

Question 4

A 0.250 kg oscillator moves with amplitude 0.0600 m and angular frequency 8.00 rad s^{-1}. At $t = 0$, it is at its positive turning point. Its potential energy is defined as zero at equilibrium.

(a) Calculate the period T. [1]

(b) Sketch separate graphs of displacement, velocity and acceleration against time over $0 \le t \le T$. Label each vertical axis with its quantity and SI unit, mark its extreme values, and mark 0, $T/4$, $T/2$, $3T/4$ and T on each time axis. [3]

(c) Calculate the total oscillation energy. [1]

(d) Sketch potential energy, kinetic energy and total energy on the same energy–displacement axes over $-0.0600 \le x \le +0.0600$ m. Label the curves and their values at equilibrium and at both turning points. [3]

Question 5

A 0.400 kg carriage is attached to a horizontal spring of spring constant 90.0 N m^{-1}. It moves without friction. Displacement x is measured from equilibrium, where the spring is unstretched. The carriage is released from rest at $x = +0.0600$ m.

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(a) Show that the carriage performs simple harmonic motion with $a = -(225\ \text{s}^{-2})x$. [3]

(b) Hence determine the angular frequency and the period. [2]

(c) Determine the maximum speed and the maximum magnitude of acceleration. [2]

(d) Calculate the velocity when the carriage first reaches $x = +0.0360$ m after release. [2]

Question 6

A light vertical spring of spring constant 98.1 N m^{-1} has its upper end fixed. A 0.250 kg object attached to its lower end is held with the spring at its natural length, then released from rest at $t = 0$. Air resistance is negligible and the spring obeys Hooke's law throughout. Let d be the equilibrium extension and let x denote displacement downwards from equilibrium. Take $g = 9.81$ m s^{-2}.

(a) Determine the equilibrium extension d. [2]

(b) Show that the object performs simple harmonic motion with $a = -(392.4\ \text{s}^{-2})x$. [2]

(c) Determine ω and the phase angle ϕ in $x = x_0 \sin(\omega t + \phi)$. Hence write down the displacement equation with numerical values. [3]

(d) Determine the second time after release at which the object is 0.0125 m below equilibrium. [3]

(e) Using the decrease in gravitational potential energy and the increase in elastic potential energy from release, calculate the kinetic energy when the object first passes through equilibrium. [2]

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Markscheme

Question 1

(a)

A: acceleration is proportional to displacement and directed towards equilibrium. **1**

B gives acceleration away from equilibrium; C has incorrect dimensions; D gives constant acceleration.

(b)

B: the first equilibrium crossing occurs at $T/4$, with negative velocity. **1**

A reverses the direction; C gives the time of the opposite turning point; D describes the velocity at that turning point rather than an equilibrium crossing.

(c)

C: $U/E = x^2/x_0^2 = 1/4$. **1**

A assumes a linear dependence on displacement; B is the kinetic-energy fraction; D is the speed divided by the maximum speed.

Question 2

(a)

Uses $T = 2\pi\sqrt{L/g}$ with $L = 0.640$ m. **1**
Obtains $T = 1.60$ s. **1**

Accept equivalent forms.

(b)

Uses $T = 2\pi\sqrt{m/k}$ with the given mass and spring constant. **1**
Obtains $T = 0.503$ s. **1**

Accept equivalent forms.

Question 3

(a)

Identifies $\omega^2 = 25.0\ \text{s}^{-2}$, giving $\omega = 5.00\ \text{rad s}^{-1}$. **1**
Uses $T = 2\pi/\omega$ to obtain $T = 1.26$ s. **1**

ECF for the period from the candidate's angular frequency.

(b)

Gives $x = (0.0800\ \text{m})\sin(\omega t)$ with $\phi = 0$ and the angular frequency from (a). **1**

Accept an equivalent cosine equation or phase angles differing by an integer multiple of 2π. ECF from (a).

(c)

Uses $\sin(\omega t) = 0.0400/0.0800 = 0.500$ and selects the first solution $\omega t = \pi/6$. **1**
Obtains $t = \pi/30$ s $= 0.105$ s. **1**

Angles are in radians. ECF from the equation in (b).

(d)

States that velocity leads displacement by $\pi/2$ rad, or by one-quarter of a cycle, OWTTE. **1**

The direction of the phase difference must be clear.

Question 4

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(a)

Uses $T = 2\pi/8.00$ to obtain $T = 0.785$ s. **1**

(b)

Draws a labelled displacement cosine curve through $+0.0600, 0, -0.0600, 0, +0.0600$ m at the specified times. **1**
Draws a labelled velocity sinusoid through $0, -0.480, 0, +0.480, 0$ m s^{-1} at the specified times. **1**
Draws a labelled acceleration sinusoid through $-3.84, 0, +3.84, 0, -3.84$ m s^{-2} at the specified times. **1**

Accept smooth qualitative curves with correctly labelled extrema and phase relationships.

(c)

Uses $E = \frac{1}{2}m\omega^2 x_0^2$ to obtain 0.0288 J. **1**

(d)

Draws the labelled potential-energy parabola with zero at equilibrium and 0.0288 J at both turning points. **1**
Draws the labelled kinetic-energy parabola with maximum 0.0288 J at equilibrium and zero at both turning points. **1**
Draws a labelled horizontal total-energy line at 0.0288 J. **1**

ECF for the energy scale from (c).

Question 5

(a)

Writes the restoring force as $F = -kx$. **1**
Uses $ma = F$ to obtain $a = -(k/m)x = -(225\ \text{s}^{-2})x$. **1**
Identifies that acceleration is proportional to displacement and opposite in direction, satisfying the condition for SHM, OWTTE. **1**

The negative sign must be retained in the derivation.

(b)

Identifies $\omega = \sqrt{225} = 15.0$ rad s^{-1}. **1**
Uses $T = 2\pi/\omega$ to obtain 0.419 s. **1**

ECF from the coefficient obtained in (a).

(c)

Uses $v_{\max} = \omega x_0$ to obtain 0.900 m s^{-1}. **1**
Uses $a_{\max} = \omega^2 x_0$ to obtain 13.5 m s^{-2}. **1**

ECF from (b).

(d)

Uses $|v| = \omega\sqrt{x_0^2 - x^2}$ with the stated displacement and amplitude. **1**
Obtains $v = -0.720$ m s^{-1}, choosing the negative sign because the carriage is moving towards equilibrium. **1**

Accept an energy-conservation method. ECF from (b).

Question 6

(a)

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Balances forces at equilibrium: $kd = mg$. **1**
Obtains $d = (0.250)(9.81)/98.1 = 0.0250$ m. **1**

(b)

Writes $F = mg - k(d + x)$ and uses $mg = kd$ to obtain $F = -kx$. **1**
Obtains $a = -(392.4\ \text{s}^{-2})x$, identifying acceleration proportional to displacement and directed towards equilibrium, OWTTE. **1**

Accept a derivation using separate equilibrium and displaced force equations.

(c)

Determines $\omega = \sqrt{392.4} = 19.8$ rad s^{-1}. **1**
Uses the initial negative turning point to identify $\phi = -\pi/2$ rad. **1**
Writes the displacement equation with amplitude 0.0250 m and the determined angular frequency and phase. **1**

Accept $\phi = 3\pi/2$ rad or equivalent phase angles. ECF for amplitude from (a) and angular frequency from (b).

(d)

Substitutes the displacement to obtain $\sin(\omega t - \pi/2) = 0.0125/0.0250 = 0.500$. **1**
Selects $\omega t - \pi/2 = 5\pi/6$ for the second occurrence, giving $\omega t = 4\pi/3$. **1**
Obtains $t = 4\pi/(3\sqrt{392.4})$ s $= 0.211$ s. **1**

Accept the equivalent cosine method with the second solution $\omega t = 4\pi/3$. Use unrounded angular frequency. ECF from (c) for a consistent second occurrence.

(e)

Applies energy conservation as $K = mgd - \frac{1}{2}kd^2$. **1**
Obtains $K = 0.0307$ J. **1**

ECF from (a). An answer using oscillation energy alone does not earn the first mark unless the gravitational and elastic energy changes are identified.

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