

Kinematics and projectile motion: study notes

IB Physics HL · A.1

Kinematics describes how a body moves without asking why: position, displacement, velocity and acceleration, and how they are linked through gradients, areas and the four equations of uniformly accelerated motion. The exam uses this topic as the opening step of longer questions: reading a velocity-time graph, solving a projectile launched from a height, or a short explain part on how air resistance changes the flight. Multiple-choice items test the same ideas. Unless a question gives a value, use the data booklet value $g = 9.81 \text{ m s}^{-2}$.

1. Position, displacement, velocity and acceleration

Position is where a body is relative to a chosen origin. Displacement s is the change in position: a vector, so it has a direction and can be negative or zero even after a long journey. Distance is the scalar length of the path actually travelled, so it never decreases. A ball thrown up and caught at the same point has zero displacement but a non-zero distance.

Velocity is the rate of change of position, $v = \frac{\Delta s}{\Delta t}$, a vector; speed is the rate of change of distance, a scalar. Acceleration is the rate of change of velocity, $a = \frac{\Delta v}{\Delta t}$, also a vector, so a body slowing down or changing direction is accelerating.

Average values use the whole interval: average velocity = $\frac{\text{total displacement}}{\text{total time}}$, average speed = $\frac{\text{total distance}}{\text{total time}}$. Instantaneous values apply at one moment and are found by shrinking the interval to zero, which on a graph means the gradient of the tangent at that instant. Always state the sign convention you are using (for example, upwards positive) before you write any equation.

2. Determining quantities from graphs

These graph rules are not in the booklet and must be learned.

Displacement-time graph: the gradient is velocity. A straight line means constant velocity; a curve of increasing steepness means increasing speed; a horizontal section means the body is at rest. The gradient of the tangent gives the instantaneous velocity; the gradient of the chord between two points gives the average velocity over that interval.

Velocity-time graph: the gradient is acceleration and the area between the line and the time axis is displacement. Area below the axis counts as negative displacement, so total distance adds the magnitudes while displacement subtracts them. A straight sloping line means uniform acceleration; a curve means non-uniform acceleration.

Acceleration-time graph: the area under the line is the change in velocity.

For a body thrown vertically upwards with upwards positive, the velocity-time graph is a straight line of gradient $-g$ that crosses the axis at the highest point, and the displacement-time graph is an inverted parabola.

3. Equations of motion for uniform acceleration

When acceleration is constant the four equations in the data booklet apply, with u the initial velocity, v the final velocity, a the acceleration, s the displacement and t the time:

$$s = \frac{(u + v)t}{2}, \quad v = u + at, \quad s = ut + \frac{1}{2}at^2, \quad v^2 = u^2 + 2as$$

Each equation omits one of the five quantities, so list what you know and what you want, then choose the equation that avoids the unknown you do not need. Every symbol is measured from the same origin and in the same positive direction, so a displacement that ends below the start is negative if upwards is positive.

Non-uniform acceleration (a changing gradient on a velocity-time graph, for example a falling object with air resistance) cannot be handled with these equations. In that case use graphical methods: tangent gradients for instantaneous values and areas, estimated by counting squares, for displacement.

Free fall near the Earth's surface is the standard case of uniform acceleration: $a = g = 9.81 \text{ m s}^{-2}$ downwards, independent of mass when air resistance is ignored.

4. Projectile motion without fluid resistance

A projectile is any body moving only under gravity after launch. The key idea is to treat the horizontal and vertical motions separately, linked only by the common time t .

Horizontal: no force, so no acceleration; the horizontal velocity stays at $u_x = u \cos \theta$ and range = $u_x t$.

Vertical: constant acceleration g downwards; the initial vertical velocity is $u_y = u \sin \theta$ and all four equations apply with $a = -9.81 \text{ m s}^{-2}$ if upwards is positive.

Three launch cases appear. Launched horizontally from a height h : $u_y = 0$, so $h = \frac{1}{2}gt^2$ fixes the time of flight, then the range follows from $u_x t$. Launched above the horizontal: the body rises until $v_y = 0$, then falls; the path is a symmetric parabola if it lands at launch height. Launched below the horizontal: u_y is negative from the start and the time of flight is shorter than for a horizontal or upward launch at the same speed from the same height.

At any instant the speed is $\sqrt{v_x^2 + v_y^2}$ and the direction is $\tan \phi = v_y/v_x$. At the highest point the velocity is purely horizontal (not zero) and the acceleration is still g . A body dropped and one projected horizontally from the same height hit the ground at the same time; a body thrown upwards and one thrown downwards at the same speed from the same height hit the ground with the same speed but at different times.

5. Qualitative effect of fluid resistance

Air (or any fluid) exerts a drag force opposite to the velocity that grows with speed. Consequences to quote in explain parts: the horizontal velocity now decreases, so the range is shorter; the maximum height is lower; the trajectory is no longer a symmetric parabola but steeper on the descent; the acceleration is no longer constant or purely vertical; the speed on landing is lower than it would be without drag (and, for a projectile that returns to launch height, lower than the launch speed); and, for a long fall, drag increases until it balances weight, the acceleration becomes zero and the body falls at a constant terminal speed. For a vertical fall, time of flight is longer than without resistance. Each of these is far more likely to be credited if you name the cause: a resistive force opposing motion whose size increases with speed.

Formulas

$s = \frac{(u+v)t}{2}$	in the booklet
$v = u + at$	in the booklet
$s = ut + \frac{1}{2}at^2$	in the booklet
$v^2 = u^2 + 2as$	in the booklet
$g = 9.81 \text{ m s}^{-2}$	in the booklet
$v = \frac{\Delta s}{\Delta t}, a = \frac{\Delta v}{\Delta t}$	learn it
gradient of s - t = velocity; gradient of v - t = acceleration; area under v - t = displacement	learn it
$u_x = u \cos \theta, u_y = u \sin \theta$; speed = $\sqrt{v_x^2 + v_y^2}$	learn it

Worked examples

Example 1. A stone is thrown vertically upwards at 15 m s^{-1} from the edge of a cliff. It strikes the sea 4.0 s later. Air resistance is negligible. Take $g = 9.8 \text{ m s}^{-2}$. (a) Calculate the maximum height reached above the cliff. (b) Determine the height of the cliff. (c) Determine the speed of the stone as it hits the sea.

1. Take upwards as positive: $u = +15 \text{ m s}^{-1}, a = -9.8 \text{ m s}^{-2}$.
2. (a) At the top $v = 0$. Use $v^2 = u^2 + 2as$: $0 = 15^2 + 2(-9.8)s$, so $s = \frac{225}{19.6} = 11.5 \text{ m}$.
3. (b) Use $s = ut + \frac{1}{2}at^2$ for the whole 4.0 s : $s = 15(4.0) + \frac{1}{2}(-9.8)(4.0)^2 = 60 - 78.4 = -18.4 \text{ m}$. The negative sign means the sea is 18.4 m below the launch point.
4. (c) Use $v = u + at$: $v = 15 + (-9.8)(4.0) = -24.2 \text{ m s}^{-1}$. The sign shows the velocity is downwards; the speed is the magnitude.

Answer: (a) 11 m (11.5 m); (b) the cliff is 18 m high; (c) impact speed 24 m s^{-1} downwards.

Example 2. A ball rolls off the edge of a horizontal table at 3.5 m s^{-1} . The table top is 1.2 m above the floor. Air resistance is negligible. Take $g = 9.8 \text{ m s}^{-2}$. (a) Show that the time taken to reach the floor is about 0.5 s . (b) Calculate the horizontal distance from the table at which the ball lands. (c) Determine the speed and direction of the ball just before it lands. (d) Explain how air resistance would change the horizontal distance.

1. (a) Vertical motion, downwards positive, $u_y = 0$: $1.2 = \frac{1}{2}(9.8)t^2$, so $t = \sqrt{\frac{2 \times 1.2}{9.8}} = 0.495 \text{ s} \approx 0.5 \text{ s}$.
2. (b) Horizontal velocity is constant: range = $u_x t = 3.5 \times 0.495 = 1.73 \text{ m}$.
3. (c) Vertical velocity at the floor: $v_y = 9.8 \times 0.495 = 4.85 \text{ m s}^{-1}$. Speed = $\sqrt{3.5^2 + 4.85^2} = \sqrt{12.25 + 23.5} = 5.98 \text{ m s}^{-1}$. Direction: $\tan \phi = \frac{4.85}{3.5}$, so $\phi = 54^\circ$ below the horizontal.
4. (d) Air resistance is a force opposing the velocity, so it has a backward horizontal component. The horizontal velocity decreases during the flight instead of staying at 3.5 m s^{-1} , so the ball travels a shorter horizontal distance in a slightly longer time.

Answer: (a) $t = 0.495 \text{ s} \approx 0.5 \text{ s}$; (b) 1.7 m ; (c) 6.0 m s^{-1} at 54° below the horizontal; (d) the range is reduced because drag decelerates the horizontal motion.

Common errors

- Mixing sign conventions: writing $u = +15$ and $a = +9.8$ for an upward throw. Decide once that one direction is positive and give every vector quantity, including the final displacement, the correct sign.
- Taking the velocity at the top of a projectile path to be zero. Only the vertical component is zero; the horizontal component is unchanged and the acceleration is still g .
- Using the uniform-acceleration equations for non-uniform motion, such as a fall with air resistance. If the velocity-time graph is curved, use tangent gradients and areas instead.
- Confusing displacement with distance when reading a velocity-time graph. Area below the time axis is negative displacement, so subtract it for displacement but add its magnitude for distance.
- Forgetting to convert units before substituting: centimetres, kilometres or milliseconds must become metres and seconds, and a speed in km h^{-1} must be converted to m s^{-1} .
- In show-that parts, writing only the rounded target. Show the substituted expression and the unrounded result, then round to the value quoted.

Before the exam, can you...

- Define displacement, velocity and acceleration as vectors, and explain the difference between average and instantaneous values?
- Read velocity from the gradient of a displacement-time graph, and acceleration and displacement from the gradient and area of a velocity-time graph?
- Choose the correct equation of motion from the booklet and apply a consistent sign convention when the motion ends below the start?
- Resolve a launch velocity into $u \cos \theta$ and $u \sin \theta$, use the vertical motion to find the time of flight and the horizontal motion to find the range, and recombine components for the impact speed and angle?
- Explain, with reference to a drag force that increases with speed, the effect of air resistance on range, height, trajectory, acceleration, impact speed and terminal speed?