

## Gravitational fields and orbits: practice worksheet

IB Physics HL · D.1 · 50 minutes · 42 marks

### Question 1

A planet of radius  $R$  has a gravitational field strength  $g_0$  at its surface. The planet may be treated as a uniform sphere.

- (a) Define gravitational field strength at a point. [1]
- (b) Find, in terms of  $g_0$ , the gravitational field strength at a height  $2R$  above the surface of the planet. [2]

### Question 2

Two small moons, A and B, move in circular orbits about the same planet. Moon A has orbital radius  $r$  and period  $T$ . Moon B has orbital radius  $4r$ .

- (a) State Kepler's third law of planetary motion. [1]
- (b) Find the orbital period of moon B in terms of  $T$ . [2]
- (c) Write down the ratio  $\frac{\text{orbital speed of B}}{\text{orbital speed of A}}$ . [1]

### Question 3

A satellite moves in a circular orbit about a planet. Data for the orbit are given below.

Orbital radius  $r = 8.0 \times 10^6$  m

Orbital period  $T = 2.0$  hours

The planet may be treated as a uniform sphere of mass  $M$ .

- (a) Show that the speed  $v$  of a satellite in a circular orbit of radius  $r$  is given by  $v = \sqrt{\frac{GM}{r}}$ . [2]
- (b) Determine the mass of the planet. [3]
- (c) Calculate the orbital speed of the satellite. [2]

### Question 4

A planet and its moon may be treated as point masses. Their centres are separated by a fixed distance  $d$ .

Mass of planet  $M_p = 5.0 \times 10^{24}$  kg

Mass of moon  $M_m = 2.0 \times 10^{23}$  kg

Separation of centres  $d = 3.0 \times 10^8$  m

Point X lies on the line joining the two centres. At X the resultant gravitational field strength is zero.

- (a) Explain why there must be a point on the line joining the centres at which the resultant gravitational field strength is zero. [2]
- (b) Determine the distance of X from the centre of the planet. [3]
- (c) Calculate the gravitational potential at X. [2]
- (d) Explain why the gravitational potential at X is not zero even though the gravitational field strength at X is zero. [1]

### Question 5

A satellite of mass  $m = 450$  kg is in a circular orbit about a planet of mass  $M = 6.0 \times 10^{24}$  kg.

Initial orbital radius  $r_1 = 7.0 \times 10^6$  m

Final orbital radius  $r_2 = 1.4 \times 10^7$  m

- (a) Show that the total energy of a satellite of mass  $m$  in a circular orbit of radius  $r$  is  $E = -\frac{GMm}{2r}$ . [2]
- (b) Calculate the energy that must be supplied to move the satellite from the orbit of radius  $r_1$  to the orbit of radius  $r_2$ . [3]
- (c) When in the lower orbit the satellite experiences a small resistive force from the upper atmosphere of the planet. Explain why the speed of the satellite increases as a result of this force. [3]

### Question 6

A small moon of a distant planet may be treated as a uniform sphere.

Mass of moon  $M = 1.2 \times 10^{23}$  kg

Radius of moon  $R = 1.5 \times 10^6$  m

A probe of mass 120 kg rests on the surface of the moon. The moon does not rotate.

- (a) Define gravitational potential at a point. [1]
- (b) Show that the escape speed from the surface of the moon is  $v_{\text{esc}} = \sqrt{\frac{2GM}{R}}$ . [2]
- (c) Calculate the escape speed from the surface of the moon. [2]
- (d) Determine the minimum energy that must be supplied to place the probe, starting from rest on the surface, into a circular orbit of radius  $2R$  about the moon. [3]
- (e) The moon has a very thin atmosphere of temperature 150 K containing hydrogen molecules of mass  $3.3 \times 10^{-27}$  kg. Estimate the root mean square speed of the hydrogen molecules. Take  $k_B = 1.38 \times 10^{-23} \text{ J K}^{-1}$ . [2]
- (f) Using your answers to (c) and (e), discuss whether the moon can retain hydrogen in its atmosphere over a long period of time. [2]

## Markscheme

### Question 1

(a)

force per unit mass on a «small/point» test mass placed at the point 1

Accept  $g = \frac{F}{m}$  with symbols defined. OWTTE.

(b)

distance from centre of planet is  $3R$  «so  $g \propto \frac{1}{(3R)^2}$ » 1

$g = \frac{g_0}{9}$  1

Using  $2R$  as the distance from the centre scores zero.

### Question 2

(a)

«square of» period is proportional to «cube of» orbital radius /  $T^2 \propto r^3$  1

Accept 'mean distance from the central body' or 'semi-major axis' in place of radius.

(b)

$\left(\frac{T_B}{T}\right)^2 = 4^3 = 64$  1

$T_B = 8T$  1

(c)

$\frac{v_B}{v_A} = \frac{1}{2}$  «since  $v \propto \frac{1}{\sqrt{r}}$ » 1

Accept 0.5.

### Question 3

(a)

gravitational force provides the centripetal force:  $\frac{GMm}{r^2} = \frac{mv^2}{r}$  1

cancels  $m$  and one  $r$  to give  $v^2 = \frac{GM}{r}$  «hence result» 1

(b)

uses  $v = \frac{2\pi r}{T}$  OR  $T^2 = \frac{4\pi^2 r^3}{GM}$  1

$M = \frac{4\pi^2 (8.0 \times 10^6)^3}{6.67 \times 10^{-11} \times (7200)^2}$  1

$M = 5.8 \times 10^{24} \text{ kg}$  1

Accept  $5.8\text{--}5.9 \times 10^{24} \text{ kg}$ . ECF for a period left in hours at the substitution line only.

(c)

$v = \frac{2\pi \times 8.0 \times 10^6}{7200}$  OR  $v = \sqrt{\frac{6.67 \times 10^{-11} \times 5.8 \times 10^{24}}{8.0 \times 10^6}}$  1

$v = 7.0 \times 10^3 \text{ m s}^{-1}$  1

ECF from (b).

### Question 4

(a)

between the bodies the field of the planet and the field of the moon act in opposite 1  
directions «along the line joining the centres»

field strength of planet decreases and that of the moon increases with distance from 1  
the planet, so at some point the magnitudes are equal «and cancel»

OWTTE.

(b)

$$\frac{GM_p}{x^2} = \frac{GM_m}{(d-x)^2} \quad 1$$

$$\frac{d-x}{x} = \sqrt{\frac{2.0 \times 10^{23}}{5.0 \times 10^{24}}} = \frac{1}{5} \quad 1$$

$$x = 2.5 \times 10^8 \text{ m} \quad 1$$

Accept equivalent forms, e.g.  $x = \frac{5}{6}d$ .

(c)

$$V = -6.67 \times 10^{-11} \left( \frac{5.0 \times 10^{24}}{2.5 \times 10^8} + \frac{2.0 \times 10^{23}}{5.0 \times 10^7} \right) \quad 1$$

$$V = -1.6 \times 10^6 \text{ J kg}^{-1} \quad 1$$

Sign of potential is implied at the substitution line but required in the final answer. ECF from (b).

(d)

potential is a scalar, so the two «negative» contributions add rather than cancel / 1

field is a vector so opposite fields can cancel

OWTTE.

### Question 5

(a)

$$\text{kinetic energy} = \frac{1}{2}mv^2 = \frac{GMm}{2r} \text{ «using } v^2 = \frac{GM}{r} \text{»} \quad 1$$

$$E = \frac{GMm}{2r} - \frac{GMm}{r} = -\frac{GMm}{2r} \quad 1$$

Potential energy  $E_p = -\frac{GMm}{r}$  must appear explicitly for the second mark.

(b)

$$\Delta E = \frac{GMm}{2} \left( \frac{1}{r_1} - \frac{1}{r_2} \right) \quad 1$$

$$\Delta E = \frac{6.67 \times 10^{-11} \times 6.0 \times 10^{24} \times 450}{2} \left( \frac{1}{7.0 \times 10^6} - \frac{1}{1.4 \times 10^7} \right) \quad 1$$

$$\Delta E = 6.4 \times 10^9 \text{ J} \quad 1$$

Accept  $6.4\text{--}6.5 \times 10^9 \text{ J}$ . Award [max 2] if the change in potential energy alone ( $1.3 \times 10^{10} \text{ J}$ ) is given.

(c)

resistive force does negative work / total energy of the satellite decreases 1

total energy  $-\frac{GMm}{2r}$  becomes more negative so the orbital radius decreases / satellite 1

spirals inwards

kinetic energy  $\frac{GMm}{2r}$  increases as  $r$  decreases, so speed increases 1

decrease in potential energy is larger than the work done against the resistive force 1

/ potential energy falls by twice the rise in kinetic energy

Award [max 3]. OWTTE.

### Question 6

(a)

work done per unit mass in bringing a «small/point» test mass from infinity to the point 1

OWTTE. Reference to infinity is required.

(b)

$$\text{total energy at surface} = \frac{1}{2}mv^2 - \frac{GMm}{R} \text{ and equals zero «at infinity with zero speed»} \quad 1$$

$$\frac{1}{2}mv^2 = \frac{GMm}{R} \text{ leading to } v^2 = \frac{2GM}{R} \quad 1$$

Accept an argument in terms of  $\frac{1}{2}v^2 = -V_g$ .

(c)

$$v_{\text{esc}} = \sqrt{\frac{2 \times 6.67 \times 10^{-11} \times 1.2 \times 10^{23}}{1.5 \times 10^6}} \quad 1$$

$$v_{\text{esc}} = 3.3 \times 10^3 \text{ m s}^{-1} \quad 1$$

(d)

$$\text{energy in orbit} = -\frac{GMm}{2(2R)} = -\frac{GMm}{4R} \quad 1$$

$$\Delta E = -\frac{GMm}{4R} - \left(-\frac{GMm}{R}\right) = \frac{3GMm}{4R} \quad 1$$

$$\Delta E = \frac{3 \times 6.67 \times 10^{-11} \times 1.2 \times 10^{23} \times 120}{4 \times 1.5 \times 10^6} = 4.8 \times 10^8 \text{ J} \quad 1$$

Award [max 2] if only the change in potential energy ( $3.2 \times 10^8 \text{ J}$ ) is calculated, i.e. the orbital kinetic energy is omitted.

(e)

$$\frac{1}{2}m\langle v^2 \rangle = \frac{3}{2}k_B T \text{ OR } v_{\text{rms}} = \sqrt{\frac{3 \times 1.38 \times 10^{-23} \times 150}{3.3 \times 10^{-27}}} \quad 1$$

$$v_{\text{rms}} = 1.4 \times 10^3 \text{ m s}^{-1} \quad 1$$

(f)

compares  $v_{\text{rms}}$  « $1.4 \times 10^3 \text{ m s}^{-1}$ » with the escape speed « $3.3 \times 10^3 \text{ m s}^{-1}$ »:  $v_{\text{rms}}$  is smaller, by a factor of only about 2.4 1

molecular speeds are distributed / some molecules have speeds greater than the escape speed, so these escape and the hydrogen is gradually lost 1

Award [max 2]. ECF from (c) and (e). A conclusion consistent with the candidate's own values is required for the second mark. OWTTE.