

Forces, momentum and circular motion: study notes

IB Physics HL · A.2

This topic is about why objects accelerate: Newton's three laws, the real forces that act on a body (normal reaction, friction, tension, weight, drag, buoyancy, springs), and two powerful consequences of the second law, namely momentum and impulse, and the centripetal force needed for circular motion. Examiners test it through a free-body diagram, a second-law calculation on an incline or connected system, a collision solved by conservation of momentum followed by a kinetic-energy check, and a circular-motion part where you must say which real forces supply the force towards the centre. The marks are mostly for correct set-up and clear reasoning, so naming forces precisely and stating conditions matters as much as the arithmetic.

1. Newton's laws and free-body diagrams

Learn the three laws in words; they are not in the booklet. First law: a body stays at rest or moves at constant velocity unless a resultant (net) force acts on it. Second law: the resultant force equals the rate of change of momentum, $F = \frac{\Delta p}{\Delta t}$, which for constant mass becomes $F = ma$. Third law: if body A exerts a force on body B, then B exerts a force on A that is equal in magnitude, opposite in direction and of the same type; the two forces act on different bodies, so they never cancel on a single free-body diagram. A free-body diagram shows one body only, as a dot or box, with every force on it drawn as an arrow from the body, labelled with its proper name: weight, normal reaction, tension, friction, drag, buoyancy. Do not draw the resultant, the acceleration or a centripetal force as separate arrows. Arrows should have lengths that reflect relative sizes where the question makes that possible (for example equal and opposite weight and normal reaction for a body at rest on a horizontal surface). Two marks are typically available: one for the complete set of forces with correct directions, one for correct labels.

2. Contact forces and field forces

Normal reaction F_N acts perpendicular to the surface, away from it. Surface friction acts parallel to the surface and opposes relative motion or the tendency to move. Static friction adjusts itself up to a maximum, $F_f \leq \mu_s F_N$ (booklet); slipping begins exactly when $F_f = \mu_s F_N$. Once sliding, dynamic friction is $F_f = \mu_d F_N$ (booklet), with $\mu_d < \mu_s$ usually. The elastic restoring force of a spring obeys Hooke's law $F_H = -kx$ (booklet), the minus sign showing it is directed towards the equilibrium position. Viscous drag on a sphere of radius r moving slowly at speed v through a fluid of viscosity η is given by Stokes' law, $F_d = 6\pi\eta rv$ (booklet), opposing the velocity; terminal speed occurs when drag plus buoyancy balance weight. Buoyancy (Archimedes) is the upward force equal to the weight of fluid displaced, $F_b = \rho Vg$ (booklet), where ρ is the fluid density and V the displaced volume. Weight is the field force $F_g = mg$ (booklet), acting vertically downward towards the centre of the Earth, with $g = 9.81 \text{ m s}^{-2}$. Tension acts along a string or rod, pulling away from the body at each end; a light inextensible string has the same tension throughout. On an incline, resolve weight into $mg \sin \theta$ down the slope and $mg \cos \theta$ into the slope, so that $F_N = mg \cos \theta$ when there is no other perpendicular force.

3. Linear momentum and impulse

Linear momentum is $p = mv$ (booklet), a vector in the direction of the velocity, unit kg m s^{-1} . Impulse is the change in momentum: $J = F\Delta t$ (booklet), and the identity $J = \Delta p$ is not printed in the booklet and must be learnt; the unit is N s , equivalent to kg m s^{-1} . On a force-time graph the impulse is the area between the line and the time axis, so a curved force-time graph is handled by counting squares

or by approximating the shape as triangles and rectangles. Newton's second law in its general form, $F = \frac{\Delta p}{\Delta t}$ (booklet), is the one to use when the mass changes (a rocket ejecting fuel, sand falling onto a belt) or when the question gives momentum rather than acceleration. Because momentum is a vector, define a positive direction before substituting: a ball of mass m hitting a wall at speed v and rebounding at speed v has $\Delta p = -2mv$ (initial direction taken as positive), not zero. A longer contact time for the same change in momentum means a smaller average force, which is why crumple zones and padded surfaces reduce injury.

4. Conservation of momentum, collisions and explosions

Learn the statement: the total momentum of a system is constant provided no resultant external force acts on it. This condition is not in the booklet and must be written down whenever you use conservation of momentum; the examiner usually looks for it. During a brief collision the internal forces between the bodies are large and the external forces (friction, air resistance) are negligible over the short time, so momentum is conserved even if friction acts before and after. In one dimension write total momentum before equals total momentum after, with signs for direction. In two dimensions apply the same equation separately to the x and y components; an explosion from rest, for example a stationary nucleus emitting a particle, gives fragments with equal and opposite momenta, and because $E_k = \frac{1}{2}mv^2 = \frac{p^2}{2m}$ (booklet, listed under A.3) the lighter fragment carries the larger share of kinetic energy. An elastic collision conserves total kinetic energy as well as momentum; an inelastic collision does not, kinetic energy being transferred to internal energy, sound and deformation. A perfectly inelastic collision is one in which the bodies stick together and move with a common velocity; this is the case of greatest kinetic-energy loss consistent with momentum conservation. Momentum is conserved in every collision in which no resultant external force acts on the system (or the external impulse is negligible during the brief contact); kinetic energy is conserved only in elastic ones. Always finish a collision calculation by comparing kinetic energies before and after, stating whether the collision is elastic or inelastic and giving the energy lost.

5. Circular motion at constant speed

A body moving in a circle of radius r at constant speed v has a continuously changing velocity direction, so it is accelerating towards the centre and needs a resultant force towards the centre. The period T is the time for one revolution, frequency $f = \frac{1}{T}$, and angular velocity is $\omega = \frac{2\pi}{T} = 2\pi f$ in rad s^{-1} . From the booklet: $v = \frac{2\pi r}{T} = \omega r$ and $a = \frac{v^2}{r} = \omega^2 r = \frac{4\pi^2 r}{T^2}$, so the centripetal force is $F = \frac{mv^2}{r} = m\omega^2 r$. The essential rule to learn: the centripetal force is not a new force; it is the resultant of the real forces acting towards the centre. On a free-body diagram you draw only the real forces (tension, weight, normal reaction, friction, gravitational attraction) and then write the equation 'resultant towards centre = $\frac{mv^2}{r}$ '. Because the force is always perpendicular to the velocity it does no work, so the speed stays constant while the velocity changes. Typical sources of centripetal force: tension for a whirled mass on a string, friction for a car on a flat bend, gravity for a satellite, the horizontal component of the normal reaction for a banked road, and the horizontal component of the tension for a conical pendulum.

6. Vertical circles, banked and horizontal turns

In a vertical circle the speed is not constant but the centripetal relation still holds at each point. For a mass on a string, at the lowest point tension and weight oppose, so $T - mg = \frac{mv^2}{r}$ and tension is greatest; at the highest point both act downward towards the centre, so $T + mg = \frac{mv^2}{r}$ and tension is

least. The string stays taut at the top only if $v \geq \sqrt{gr}$; the minimum speed $v = \sqrt{gr}$ is the limiting case $T = 0$. For a car over a hump the same logic applies with normal reaction: $mg - F_N = \frac{mv^2}{r}$, and the car leaves the road when $F_N = 0$. On a horizontal (flat) turn friction supplies the whole centripetal force, so the maximum speed without skidding comes from $\mu_s F_N = \mu_s mg = \frac{mv^2}{r}$, giving $v_{max} = \sqrt{\mu_s gr}$, independent of mass. On a banked turn of angle θ with no friction, the normal reaction is tilted: vertically $F_N \cos \theta = mg$, horizontally $F_N \sin \theta = \frac{mv^2}{r}$, so $\tan \theta = \frac{v^2}{gr}$ at the design speed. Faster than this, friction must act down the slope; slower, friction acts up the slope. Writing the resolving equations in this two-line form typically earns the method marks even if a later arithmetic step goes wrong.

Formulas

$F_f \leq \mu_s F_N, \quad F_f = \mu_d F_N$	in the booklet
$F_H = -kx$	in the booklet
$F_d = 6\pi\eta rv$	in the booklet
$F_b = \rho Vg, \quad F_g = mg$	in the booklet
$p = mv, \quad J = F\Delta t, \quad F = ma = \frac{\Delta p}{\Delta t}$	in the booklet
$E_k = \frac{1}{2}mv^2 = \frac{p^2}{2m}$	in the booklet (under A.3)
$a = \frac{v^2}{r} = \omega^2 r = \frac{4\pi^2 r}{T^2}, \quad v = \frac{2\pi r}{T} = \omega r$	in the booklet
Newton's three laws in words; third-law pairs act on different bodies	learn it
Impulse equals change of momentum: $J = \Delta p$	learn it
Momentum conserved only if no resultant external force acts on the system	learn it
Elastic: total E_k conserved; inelastic: E_k not conserved	learn it
Centripetal force is the resultant of real forces, never an extra arrow	learn it
Slipping starts when $F_f = \mu_s F_N$; flat bend $v_{max} = \sqrt{\mu_s gr}$; bank $\tan \theta = \frac{v^2}{gr}$	learn it
Top of vertical circle $T + mg = \frac{mv^2}{r}$; bottom $T - mg = \frac{mv^2}{r}$	learn it

Worked examples

Example 1. A crate of mass 2.0 kg slides down a rough slope inclined at 30° to the horizontal. The coefficient of dynamic friction between crate and slope is 0.20. (a) Draw and label the free-body diagram for the crate. (b) Show that the normal reaction is about 17 N. (c) Calculate the acceleration of the crate down the slope.

- (a) Three forces on the crate: weight mg vertically downward, normal reaction F_N perpendicular to the slope and away from it, friction F_f parallel to the slope pointing up the slope (opposing the sliding).
- (b) Perpendicular to the slope there is no acceleration, so $F_N = mg \cos \theta = 2.0 \times 9.81 \times \cos 30^\circ = 16.99 \text{ N} \approx 17 \text{ N}$.
- (c) Friction while sliding: $F_f = \mu_d F_N = 0.20 \times 16.99 = 3.40 \text{ N}$.
- Component of weight down the slope: $mg \sin \theta = 2.0 \times 9.81 \times 0.50 = 9.81 \text{ N}$.

5. Newton's second law along the slope: $ma = mg \sin \theta - F_f = 9.81 - 3.40 = 6.41 \text{ N}$, so $a = \frac{6.41}{2.0} = 3.2 \text{ m s}^{-2}$.

Answer: Normal reaction $\approx 17 \text{ N}$; acceleration 3.2 m s^{-2} down the slope.

Example 2. Trolley A of mass 0.80 kg moves at 3.0 m s^{-1} on a frictionless horizontal track and collides with stationary trolley B of mass 1.2 kg . The trolleys couple together. (a) State the condition for the total momentum of the trolleys to be conserved. (b) Show that the speed of the coupled trolleys is 1.2 m s^{-1} . (c) Determine the kinetic energy lost in the collision and state the type of collision.

- (a) Momentum is conserved because no resultant external force acts on the two-trolley system during the collision (the track is frictionless and the internal forces between the trolleys are equal and opposite).
- (b) Take the initial direction of A as positive. Momentum before: $p = 0.80 \times 3.0 + 1.2 \times 0 = 2.4 \text{ kg m s}^{-1}$.
- After coupling the combined mass is $0.80 + 1.2 = 2.0 \text{ kg}$, so $2.4 = 2.0v$ and $v = 1.2 \text{ m s}^{-1}$.
- (c) Kinetic energy before: $E_k = \frac{1}{2} \times 0.80 \times 3.0^2 = 3.6 \text{ J}$.
- Kinetic energy after: $E_k = \frac{1}{2} \times 2.0 \times 1.2^2 = 1.44 \text{ J}$.
- Energy lost $= 3.6 - 1.44 = 2.16 \text{ J} \approx 2.2 \text{ J}$, transferred to internal energy and sound. Kinetic energy is not conserved, so the collision is inelastic (perfectly inelastic, since they stick).

Answer: Common speed 1.2 m s^{-1} ; about 2.2 J of kinetic energy is lost; the collision is (perfectly) inelastic.

Example 3. A ball of mass 0.25 kg is attached to a light string of length 0.60 m and swung in a vertical circle. At the highest point its speed is 3.0 m s^{-1} . (a) Identify the forces that provide the centripetal force at the highest point. (b) Calculate the tension in the string at the highest point. (c) Determine the minimum speed at the highest point for which the string remains taut.

- (a) At the top both the weight of the ball and the tension in the string act vertically downward, towards the centre of the circle. The centripetal force is the resultant of these two real forces:
 $T + mg = \frac{mv^2}{r}$.
- (b) Centripetal acceleration: $a = \frac{v^2}{r} = \frac{3.0^2}{0.60} = 15 \text{ m s}^{-2}$.
- Required resultant: $ma = 0.25 \times 15 = 3.75 \text{ N}$. Weight: $mg = 0.25 \times 9.81 = 2.45 \text{ N}$.
- Tension: $T = 3.75 - 2.45 = 1.30 \text{ N} \approx 1.3 \text{ N}$.
- (c) The string is just taut when $T = 0$, so weight alone supplies the centripetal force: $mg = \frac{mv^2}{r}$, giving $v = \sqrt{gr} = \sqrt{9.81 \times 0.60} = \sqrt{5.89} = 2.4 \text{ m s}^{-1}$.

Answer: Tension at the top $\approx 1.3 \text{ N}$; minimum speed at the top 2.4 m s^{-1} .

Common errors

- Drawing a 'centrifugal force' or a separate 'centripetal force' arrow on a free-body diagram. Draw only the real forces (tension, weight, normal reaction, friction) and then state that their resultant towards the centre provides the centripetal force.

- Getting the direction of friction wrong, especially on inclines and in connected systems. Friction opposes the relative motion of the surfaces, or the tendency to move; decide which way the body would slide without friction, then draw friction the opposite way.
- Using conservation of momentum without stating the condition. Always write that momentum is conserved because there is no resultant external force on the system during the collision; this is often a mark in its own right.
- Assuming kinetic energy is conserved in every collision. Only elastic collisions conserve kinetic energy; in a sticking collision calculate E_k before and after and show the loss explicitly.
- Subtracting weight at the top of a vertical circle. At the highest point weight and tension (or normal reaction) both point towards the centre, so they add: $T + mg = \frac{mv^2}{r}$. Subtraction applies only at the lowest point.
- Forgetting signs in a rebound and computing $\Delta p = m(v - u)$ with both speeds taken as positive, which wrongly gives zero for a rebound at the same speed. Choose a positive direction, write velocities with signs, and compute $\Delta p = mv - mu$; with u and v the speeds before and after, the magnitude is $m(u + v)$, which is $2mv$ for a rebound at the same speed.

Before the exam, can you...

- State Newton's three laws in words, and explain why a third-law pair never appears on the same free-body diagram?
- Draw a complete, correctly labelled free-body diagram for a body on an incline, in a vertical circle and on a banked turn, with friction in the right direction?
- Resolve forces along and perpendicular to a slope and apply $F = ma$ to a single body or a connected system, using $F_N = mg \cos \theta$ and $F_f = \mu F_N$?
- Solve a one- or two-dimensional collision or explosion with conservation of momentum, state the no-external-force condition, and check kinetic energy to classify it as elastic or inelastic?
- Write 'resultant of real forces towards the centre = $\frac{mv^2}{r}$ ', for a car on a flat or banked bend, a mass on a string at the top and bottom of a vertical circle, and find a minimum or maximum speed condition?