

Mathematical equations

Triangle; circle		
$A = \frac{1}{2}(bh)$ (b base, h height)	$A = \pi r^2$ (r radius)	$C = 2\pi r$
Cuboid; prism		
$V = lwh$ (l length, w width, h height)	$V = Ah$ (A area of cross-section)	
Sphere; cylinder	$V = \frac{4}{3}\pi r^3$	$V = \pi r^2 h$ $A = 2\pi rh$
Vector components	$A_H = A \cos \theta$	$A_V = A \sin \theta$
Trigonometry	$\tan \theta = \frac{\sin \theta}{\cos \theta}$	$\sin^2 \theta + \cos^2 \theta = 1$

Uncertainties

Sum or difference	If $y = a \pm b$, then $\Delta y = \Delta a + \Delta b$
Product or quotient	If $y = \frac{ab}{c}$, then $\frac{\Delta y}{y} = \frac{\Delta a}{a} + \frac{\Delta b}{b} + \frac{\Delta c}{c}$
Power	If $y = a^n$, then $\frac{\Delta y}{y} = \left n \frac{\Delta a}{a} \right $

Metric (SI) multipliers

peta	tera	giga	mega	kilo	hecto	deca
$P 10^{15}$	$T 10^{12}$	$G 10^9$	$M 10^6$	$k 10^3$	$h 10^2$	$da 10^1$
deci	centi	milli	micro	nano	pico	femto
$d 10^{-1}$	$c 10^{-2}$	$m 10^{-3}$	$\mu 10^{-6}$	$n 10^{-9}$	$p 10^{-12}$	$f 10^{-15}$

Unit conversions

1 radian (rad) $\equiv \frac{180^\circ}{\pi}$
Temperature (K) = temperature ($^\circ\text{C}$) + 273
1 light year (ly) = 9.46×10^{15} m 1 parsec (pc) = 3.26 ly
1 astronomical unit (AU) = 1.50×10^{11} m
1 kilowatt-hour (kWh) = 3.60×10^6 J

$hc = 1.99 \times 10^{-25} \text{ J m} = 1.24 \times 10^{-6} \text{ eV m}$

Fundamental constants

Free-fall acceleration	$g \ 9.8 \text{ m s}^{-2}$ (Earth's surface)
Gravitational constant	$G \ 6.67 \times 10^{-11} \text{ N m}^2 \text{ kg}^{-2}$
Avogadro constant	$N_A \ 6.02 \times 10^{23} \text{ mol}^{-1}$
Gas constant	$R \ 8.31 \text{ J K}^{-1} \text{ mol}^{-1}$
Boltzmann constant	$k_B \ 1.38 \times 10^{-23} \text{ J K}^{-1}$
Stefan–Boltzmann constant	$\sigma \ 5.67 \times 10^{-8} \text{ W m}^{-2} \text{ K}^{-4}$
Coulomb constant	$k \ 8.99 \times 10^9 \text{ N m}^2 \text{ C}^{-2}$
Permittivity of free space	$\epsilon_0 \ 8.85 \times 10^{-12} \text{ C}^2 \text{ N}^{-1} \text{ m}^{-2}$
Permeability of free space	$\mu_0 \ 4\pi \times 10^{-7} \text{ T m A}^{-1}$
Speed of light	$c \ 3.00 \times 10^8 \text{ m s}^{-1}$
Planck constant	$h \ 6.63 \times 10^{-34} \text{ J s}$
Elementary charge	$e \ 1.60 \times 10^{-19} \text{ C}$
Electron rest mass	$m_e \ 9.110 \times 10^{-31} \text{ kg} = 0.000549 \text{ u} = 0.511 \text{ MeV } c^{-2}$

Proton rest mass	$m_p \ 1.673 \times 10^{-27} \text{ kg} = 1.007276 \text{ u} = 938 \text{ MeV } c^{-2}$
Neutron rest mass	$m_n \ 1.675 \times 10^{-27} \text{ kg} = 1.008665 \text{ u} = 940 \text{ MeV } c^{-2}$
Atomic mass unit	$u \ 1.661 \times 10^{-27} \text{ kg} = 931.5 \text{ MeV } c^{-2}$
Solar constant	$S \ 1.36 \times 10^3 \text{ W m}^{-2}$
Fermi radius	$R_0 \ 1.20 \times 10^{-15} \text{ m}$

A. Space, time and motion

All equations give magnitudes only: vector notation is not used.
A.1 KINEMATICS

SUVAT $s = \frac{u+v}{2}t \quad v = u + at \quad s = ut + \frac{1}{2}at^2 \quad v^2 = u^2 + 2as$

A.2 FORCES AND MOMENTUM

Static friction	$F_f \leq \mu_s F_N$	Dynamic friction	$F_f = \mu_d F_N$
Hooke's law	$F_H = -kx$	Stokes' drag	$F_d = 6\pi\eta rv$
Buoyancy	$F_b = \rho Vg$	Weight	$F_g = mg$
Momentum	$p = mv$	Impulse	$J = F\Delta t$

Newton's second law $F = ma = \frac{\Delta p}{\Delta t}$

Circular motion $a = \frac{v^2}{r} = \omega^2 r = \frac{4\pi^2 r}{T^2} \quad v = \frac{2\pi r}{T} = \omega r$

A.3 WORK, ENERGY AND POWER

Work	$W = Fs \cos \theta$	Power	$P = \frac{\Delta W}{\Delta t} = Fv$
Kinetic energy	$E_k = \frac{1}{2}mv^2 = \frac{p^2}{2m}$	Change in GPE	$\Delta E_p = mg\Delta h$
Elastic PE	$E_H = \frac{1}{2}k\Delta x^2$		
Efficiency	$\eta = \frac{\text{useful work out}}{\text{total work in}} = \frac{\text{useful power out}}{\text{total power in}}$		

A.4 RIGID BODY MECHANICS

 Torque	$\tau = Fr \sin \theta$	 Moment of inertia	$I = \Sigma mr^2$
	$\Delta \theta = \frac{\omega_f + \omega_i}{2}t \quad \omega_f = \omega_i + \alpha t$		
 Rotational SUVAT	$\Delta \theta = \omega_i t + \frac{1}{2}\alpha t^2 \quad \omega_f^2 = \omega_i^2 + 2\alpha\Delta\theta$		
 Newton II (rotation)	$\tau = I\alpha$	 Angular momentum	$L = I\omega$
 Angular impulse	$\Delta L = \tau\Delta t \quad \Delta L = \Delta(I\omega)$		
 Rotational KE	$E_k = \frac{1}{2}I\omega^2 = \frac{L^2}{2I}$		

A.5 GALILEAN AND SPECIAL RELATIVITY

HL Galilean transformation	$x' = x - vt \quad t' = t \quad u' = u - v$
HL Lorentz transformation	$x' = \gamma(x - vt) \quad \text{where } \gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} \quad t' = \gamma\left(t - \frac{vx}{c^2}\right)$
HL Relativistic velocity	$u' = \frac{u - v}{1 - \frac{uv}{c^2}}$
HL Spacetime interval	$(\Delta s)^2 = (c\Delta t)^2 - \Delta x^2$
HL Time dilation	$\Delta t = \gamma\Delta t_0$
HL Length contraction	$L = \frac{L_0}{\gamma}$

 Spacetime axes angle $\tan \theta = \frac{v}{c}$

B. The particulate nature of matter

B.1 THERMAL ENERGY TRANSFERS

Density	$\rho = \frac{m}{V}$	Mean particle KE	$\overline{E_k} = \frac{3}{2}k_B T$
Specific heat	$Q = mc\Delta T$	Latent heat	$Q = mL$
Conduction	$\frac{\Delta Q}{\Delta t} = -kA \frac{\Delta T}{\Delta x}$	Luminosity	$L = \sigma AT^4$
Apparent brightness	$b = \frac{L}{4\pi d^2}$	Wien's law	$\lambda_{\text{max}} T = 2.9 \times 10^{-3} \text{ m K}$

B.2 GREENHOUSE EFFECT

Emissivity emissivity = $\frac{\text{power radiated per unit area}}{\sigma T^4}$

Albedo albedo = $\frac{\text{total scattered power}}{\text{total incident power}}$

B.3 GAS LAWS

Pressure	$P = \frac{F}{A}$	Moles	$n = \frac{N}{N_A}$
Fixed amount of gas	$\frac{PV}{T} = \text{constant}$	Ideal gas	$PV = nRT = Nk_B T$
Kinetic theory	$P = \frac{1}{3}\rho v^2$	Internal energy (monatomic)	$U = \frac{3}{2}nRT = \frac{3}{2}Nk_B T$

B.4 THERMODYNAMICS

 First law	$Q = \Delta U + W$	 Work, constant P	$W = P\Delta V$
 Change in internal energy (monatomic)	$\Delta U = \frac{3}{2}nR\Delta T = \frac{3}{2}Nk_B\Delta T$		
 Entropy change	$\Delta S = \frac{\Delta Q}{T}$	 Entropy	$S = k_B \ln \Omega$
 Adiabatic (monatomic)	$PV^{5/3} = \text{constant}$	 Carnot	$\eta_{\text{Carnot}} = 1 - \frac{T_c}{T_h}$
 Heat engine efficiency	$\eta = \frac{\text{useful work}}{\text{input energy}}$		

B.5 CURRENT AND CIRCUITS

Current	$I = \frac{\Delta q}{\Delta t}$	Potential difference	$V = \frac{W}{q}$
Resistance	$R = \frac{V}{I}$	Resistivity	$\rho = \frac{RA}{L}$
Power	$P = IV = I^2 R = \frac{V^2}{R}$	EMF	$\mathcal{E} = I(R + r)$
Series	$I = I_1 = I_2 = \dots \quad V = V_1 + V_2 + \dots \quad R_s = R_1 + R_2 + \dots$		
Parallel	$I = I_1 + I_2 + \dots \quad V = V_1 = V_2 = \dots \quad \frac{1}{R_p} = \frac{1}{R_1} + \frac{1}{R_2} + \dots$		

C. Wave behaviour

C.1 SIMPLE HARMONIC MOTION

SHM	$a = -\omega^2 x$	Period	$T = \frac{1}{f} = \frac{2\pi}{\omega}$
Mass on spring	$T = 2\pi\sqrt{\frac{m}{k}}$	Pendulum	$T = 2\pi\sqrt{\frac{L}{g}}$
 SHM x and v	$x = x_0 \sin(\omega t + \phi) \quad v = \omega x_0 \cos(\omega t + \phi)$		
 SHM speed at x	$v = \pm \omega \sqrt{x_0^2 - x^2}$		
 Total energy	$E_T = \frac{1}{2}m\omega^2 x_0^2$	 Potential energy	$E_p = \frac{1}{2}m\omega^2 x^2$

C.2 WAVE MODEL C.3 WAVE PHENOMENA

Wave equation	$v = f\lambda = \frac{\lambda}{T}$	Double-slit	$s = \frac{\lambda D}{d}$
Snell's law	$\frac{n_1}{n_2} = \frac{\sin \theta_2}{\sin \theta_1} = \frac{v_2}{v_1}$		
Constructive; destructive	path difference = $n\lambda$	path difference = $\left(n + \frac{1}{2}\right)\lambda$	
 Single slit, first min.	$\theta = \frac{\lambda}{b}$	 Grating	$n\lambda = d \sin \theta$
C.5 DOPPLER EFFECT			
Light ($v \ll c$)	$\frac{\Delta f}{f} = \frac{\Delta \lambda}{\lambda} \approx \frac{v}{c}$		
 Moving source	$f' = f\left(\frac{v}{v \pm u_s}\right)$	 Moving observer	$f' = f\left(\frac{v \pm u_o}{v}\right)$

D. Fields

D.1 GRAVITATIONAL FIELDS

Gravitation	$F = G \frac{m_1 m_2}{r^2}$	Field strength	$g = \frac{F}{m} = G \frac{M}{r^2}$
 Potential energy	$E_p = -G \frac{m_1 m_2}{r}$	 Potential	$V_g = -G \frac{M}{r}$
 Potential gradient	$g = -\frac{\Delta V_g}{\Delta r}$	 Work done	$W = m\Delta V_g$
 Escape speed	$v_{\text{esc}} = \sqrt{\frac{2GM}{r}}$	 Orbital speed	$v_{\text{orbital}} = \sqrt{\frac{GM}{r}}$

D.2 ELECTRIC AND MAGNETIC FIELDS

Coulomb's law	$F = k \frac{q_1 q_2}{r^2}$ where $k = \frac{1}{4\pi\epsilon_0}$		
Field strength	$E = \frac{F}{q}$	Parallel plates	$E = \frac{V}{d}$
 Potential energy	$E_p = k \frac{q_1 q_2}{r}$	 Potential	$V_e = \frac{kQ}{r}$
 Potential gradient	$E = -\frac{\Delta V_e}{\Delta r}$	 Work done	$W = q\Delta V_e$

D.3 MOTION IN ELECTROMAGNETIC FIELDS

On a charge	$F = qvB \sin \theta$	On a wire	$F = BIL \sin \theta$
Parallel wires	$\frac{F}{L} = \mu_0 \frac{I_1 I_2}{2\pi r}$		

D.4 INDUCTION

 Magnetic flux	$\Phi = BA \cos \theta$	 Faraday, Lenz	$\mathcal{E} = -N \frac{\Delta \Phi}{\Delta t}$
 Moving conductor	$\mathcal{E} = BvL$		

E. Nuclear and quantum physics

E.1 STRUCTURE OF THE ATOM

Photon energy	$E = hf$	 Nuclear radius	$R = R_0 A^{1/3}$
 Hydrogen levels	$E = -\frac{13.6}{n^2} \text{ eV}$	 Bohr condition	$mvr = \frac{nh}{2\pi}$

E.2 QUANTUM PHYSICS

 Photo-electric	$E_{\text{max}} = hf - \Phi$	 de Broglie	$\lambda = \frac{h}{p}$
 Compton scattering	$\lambda_f - \lambda_i = \Delta \lambda = \frac{h}{m_e c} (1 - \cos \theta)$		

E.3 RADIOACTIVE DECAY E.5 FUSION AND STARS

Mass-energy	$E = mc^2$	 Decay law	$N = N_0 e^{-\lambda t}$
 Activity	$A = \lambda N = \lambda N_0 e^{-\lambda t}$	 Half-life	$T_{1/2} = \frac{\ln 2}{\lambda}$
Stellar parallax	$d(\text{parsec}) = \frac{1}{p(\text{arc-second})}$		