# IB Maths: Applications and Interpretation HL *one-page formula sheet*

HL higher level only pietromeloni.com

## Prior learning

Parallelogram $A = bh$ ($b$ base, $h$ height)

Triangle $A = \frac{1}{2}(bh)$ ($b$ base, $h$ height)

Trapezoid $A = \frac{1}{2}(a+b)h$ ($a$, $b$ parallel sides, $h$ height)

Circle $A = \pi r^2$ ($r$ radius)

Circumference $C = 2\pi r$ ($r$ radius)

Cuboid $V = lwh$ ($l$ length, $w$ width, $h$ height)

Cylinder $V = \pi r^2 h$ ($r$ radius, $h$ height)

Prism $V = Ah$ ($A$ area of cross-section, $h$ height)

Cylinder, curved surface $A = 2\pi rh$ ($r$ radius, $h$ height)

Distance (2D) $d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$

Midpoint (2D) $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$

HL Quadratic formula

$$ax^2 + bx + c = 0 \Rightarrow x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}, \quad a \neq 0$$

## Topic 1: Number and algebra

1.2 Arithmetic: $n$th term $u_n = u_1 + (n-1)d$

Arithmetic: sum $S_n = \frac{n}{2}(2u_1 + (n-1)d); \quad S_n = \frac{n}{2}(u_1 + u_n)$

1.3 Geometric: $n$th term $u_n = u_1 r^{n-1}$

Geometric: sum $S_n = \frac{u_1(r^n - 1)}{r - 1} = \frac{u_1(1 - r^n)}{1 - r}, \quad r \neq 1$

1.4 Compound interest

$$FV = PV \times \left(1 + \frac{r}{100k}\right)^{kn}$$

$FV$ future value, $PV$ present value, $n$ number of years, $k$ compounding periods per year, $r$% nominal annual rate of interest

1.5 Exponents, logs

$$a^x = b \iff x = \log_a b, \quad a > 0,\ b > 0,\ a \neq 1$$

1.6 Percentage error $\varepsilon = \left|\frac{v_A - v_E}{v_E}\right| \times 100\%$

$v_E$ exact value, $v_A$ approximate value of $v$

HL 1.9 Log laws

$$\log_a xy = \log_a x + \log_a y \qquad \log_a \frac{x}{y} = \log_a x - \log_a y$$

$$\log_a x^m = m \log_a x \qquad \text{for } a, x, y > 0$$

HL 1.11 Geometric: infinite sum $S_\infty = \frac{u_1}{1 - r}, \quad |r| < 1$

HL 1.12 Complex numbers $z = a + bi$

HL Discriminant $\Delta = b^2 - 4ac$

HL 1.13 Polar, Euler form $z = r(\cos\theta + i\sin\theta) = re^{i\theta} = r\operatorname{cis}\theta$

HL 1.14 Determinant $2 \times 2$ $\boldsymbol{A} = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \Rightarrow \det \boldsymbol{A} = |\boldsymbol{A}| = ad - bc$

HL Inverse $2 \times 2$

$$\boldsymbol{A} = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \Rightarrow \boldsymbol{A}^{-1} = \frac{1}{\det \boldsymbol{A}}\begin{pmatrix} d & -b \\ -c & a \end{pmatrix}, \quad ad \neq bc$$

HL 1.15 Diagonalisable matrix power

$$\boldsymbol{M}^n = \boldsymbol{P}\boldsymbol{D}^n\boldsymbol{P}^{-1}$$

$\boldsymbol{P}$ matrix of eigenvectors, $\boldsymbol{D}$ diagonal matrix of eigenvalues

## Topic 2: Functions

2.1 Straight line

$$y = mx + c; \quad ax + by + d = 0; \quad y - y_1 = m(x - x_1)$$

Gradient $m = \frac{y_2 - y_1}{x_2 - x_1}$

2.5 Axis of symmetry

$$f(x) = ax^2 + bx + c \Rightarrow \text{axis of symmetry } x = -\frac{b}{2a}$$

HL 2.9 Logistic function $f(x) = \frac{L}{1 + Ce^{-kx}}, \quad L, k, C > 0$

## Topic 3: Geometry and trigonometry

3.1 Distance (3D) $d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2}$

Midpoint (3D) $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}, \frac{z_1 + z_2}{2}\right)$

Right pyramid $V = \frac{1}{3}Ah$ ($A$ area of the base, $h$ height)

Right cone $V = \frac{1}{3}\pi r^2 h$ ($r$ radius, $h$ height)

Cone, curved surface $A = \pi rl$ ($r$ radius, $l$ slant height)

Sphere volume $V = \frac{4}{3}\pi r^3$ ($r$ radius)

Sphere surface $A = 4\pi r^2$ ($r$ radius)

3.2 Sine rule $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

Cosine rule $c^2 = a^2 + b^2 - 2ab\cos C; \quad \cos C = \frac{a^2 + b^2 - c^2}{2ab}$

Triangle $A = \frac{1}{2}ab\sin C$

3.4 Arc length (degrees) $l = \frac{\theta}{360} \times 2\pi r$ ($\theta$ angle in degrees, $r$ radius)

Sector area (degrees) $A = \frac{\theta}{360} \times \pi r^2$ ($\theta$ angle in degrees, $r$ radius)

HL 3.7 Arc length (radians) $l = r\theta$ ($r$ radius, $\theta$ angle in radians)

HL Sector area (radians) $A = \frac{1}{2}r^2\theta$ ($r$ radius, $\theta$ angle in radians)

HL 3.8 Identities $\cos^2\theta + \sin^2\theta = 1 \qquad \tan\theta = \frac{\sin\theta}{\cos\theta}$

HL 3.9 Transformation matrices

$\begin{pmatrix} \cos 2\theta & \sin 2\theta \\ \sin 2\theta & -\cos 2\theta \end{pmatrix}$ reflection in the line $y = (\tan\theta)x$

$\begin{pmatrix} k & 0 \\ 0 & k \end{pmatrix}$ enlargement, scale factor $k$, centre $(0, 0)$

$\begin{pmatrix} k & 0 \\ 0 & 1 \end{pmatrix}$ horizontal stretch (parallel to the $x$-axis), scale factor $k$

$\begin{pmatrix} 1 & 0 \\ 0 & k \end{pmatrix}$ vertical stretch (parallel to the $y$-axis), scale factor $k$

$\begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$ anticlockwise rotation by $\theta$ about the origin ($\theta > 0$)

$\begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$ clockwise rotation by $\theta$ about the origin ($\theta > 0$)

HL 3.10 Magnitude $|\boldsymbol{v}| = \sqrt{v_1^2 + v_2^2 + v_3^2}, \quad \text{where } \boldsymbol{v} = \begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix}$

HL 3.11 Line: vector $\boldsymbol{r} = \boldsymbol{a} + \lambda\boldsymbol{b}$

HL Line: parametric $x = x_0 + \lambda l, \quad y = y_0 + \lambda m, \quad z = z_0 + \lambda n$

HL 3.13 Scalar product

$$\boldsymbol{v} \cdot \boldsymbol{w} = v_1w_1 + v_2w_2 + v_3w_3, \quad \text{where } \boldsymbol{v} = \begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix}, \boldsymbol{w} = \begin{pmatrix} w_1 \\ w_2 \\ w_3 \end{pmatrix}$$

$\boldsymbol{v} \cdot \boldsymbol{w} = |\boldsymbol{v}|\,|\boldsymbol{w}|\cos\theta$ ($\theta$ angle between $\boldsymbol{v}$ and $\boldsymbol{w}$)

HL Angle between vectors $\cos\theta = \frac{v_1w_1 + v_2w_2 + v_3w_3}{|\boldsymbol{v}|\,|\boldsymbol{w}|}$

HL Vector product

$$\boldsymbol{v} \times \boldsymbol{w} = \begin{pmatrix} v_2w_3 - v_3w_2 \\ v_3w_1 - v_1w_3 \\ v_1w_2 - v_2w_1 \end{pmatrix}, \quad \text{where } \boldsymbol{v} = \begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix}, \boldsymbol{w} = \begin{pmatrix} w_1 \\ w_2 \\ w_3 \end{pmatrix}$$

$|\boldsymbol{v} \times \boldsymbol{w}| = |\boldsymbol{v}|\,|\boldsymbol{w}|\sin\theta$ ($\theta$ angle between $\boldsymbol{v}$ and $\boldsymbol{w}$)

HL Parallelogram $A = |\boldsymbol{v} \times \boldsymbol{w}|$ ($\boldsymbol{v}$, $\boldsymbol{w}$ two adjacent sides)

## Topic 4: Statistics and probability

4.2 Interquartile range $\mathrm{IQR} = Q_3 - Q_1$

4.3 Mean $\bar{x}$ $\bar{x} = \frac{\sum_{i=1}^{k} f_i x_i}{n}, \quad \text{where } n = \sum_{i=1}^{k} f_i$

4.5 Probability $\mathrm{P}(A) = \frac{n(A)}{n(U)}$

Complementary $\mathrm{P}(A) + \mathrm{P}(A') = 1$

4.6 Combined $\mathrm{P}(A \cup B) = \mathrm{P}(A) + \mathrm{P}(B) - \mathrm{P}(A \cap B)$

Mutually exclusive $\mathrm{P}(A \cup B) = \mathrm{P}(A) + \mathrm{P}(B)$

Conditional $\mathrm{P}(A \mid B) = \frac{\mathrm{P}(A \cap B)}{\mathrm{P}(B)}$

Independent $\mathrm{P}(A \cap B) = \mathrm{P}(A)\,\mathrm{P}(B)$

4.7 Expected value (discrete) $\mathrm{E}(X) = \sum x\,\mathrm{P}(X = x)$

4.8 Binomial $X \sim \mathrm{B}(n, p)$ Mean $\mathrm{E}(X) = np$; Variance $\mathrm{Var}(X) = np(1 - p)$

HL 4.14 Linear transformation $\mathrm{E}(aX + b) = a\mathrm{E}(X) + b$; $\mathrm{Var}(aX + b) = a^2\,\mathrm{Var}(X)$

HL Linear combinations of independent $X_1, \ldots, X_n$

$$\mathrm{E}(a_1X_1 \pm a_2X_2 \pm \cdots \pm a_nX_n) = a_1\mathrm{E}(X_1) \pm a_2\mathrm{E}(X_2) \pm \cdots \pm a_n\mathrm{E}(X_n)$$

$$\mathrm{Var}(a_1X_1 \pm a_2X_2 \pm \cdots \pm a_nX_n) = a_1^2\,\mathrm{Var}(X_1) + a_2^2\,\mathrm{Var}(X_2) + \cdots + a_n^2\,\mathrm{Var}(X_n)$$

HL Unbiased estimate of population variance $s_{n-1}^2 = \frac{n}{n-1}s_n^2$

HL 4.17 Poisson $X \sim \mathrm{Po}(m)$ Mean $\mathrm{E}(X) = m$; Variance $\mathrm{Var}(X) = m$

HL 4.19 Transition matrices $\boldsymbol{T}^n\boldsymbol{s}_0 = \boldsymbol{s}_n$ ($\boldsymbol{s}_0$ initial state)

## Topic 5: Calculus

5.3 Derivative of $x^n$ $f(x) = x^n \Rightarrow f'(x) = nx^{n-1}$

5.5 Integral of $x^n$ $\int x^n\,dx = \frac{x^{n+1}}{n+1} + C, \quad n \neq -1$

Area, $f(x) > 0$ $A = \int_a^b y\,dx$

5.8 Trapezoidal rule

$$\int_a^b y\,dx \approx \frac{1}{2}h\big((y_0 + y_n) + 2(y_1 + y_2 + \cdots + y_{n-1})\big), \quad \text{where } h = \frac{b - a}{n}$$

HL 5.9 Derivative of $\sin x$ $f(x) = \sin x \Rightarrow f'(x) = \cos x$

HL Derivative of $\cos x$ $f(x) = \cos x \Rightarrow f'(x) = -\sin x$

HL Derivative of $\tan x$ $f(x) = \tan x \Rightarrow f'(x) = \frac{1}{\cos^2 x}$

HL Derivative of $e^x$ $f(x) = e^x \Rightarrow f'(x) = e^x$

HL Derivative of $\ln x$ $f(x) = \ln x \Rightarrow f'(x) = \frac{1}{x}$

HL Chain rule $y = g(u)$, where $u = f(x) \Rightarrow \frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$

HL Product rule $y = uv \Rightarrow \frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$

HL Quotient rule $y = \frac{u}{v} \Rightarrow \frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$

HL 5.11 Standard integrals

$$\int \frac{1}{x}\,dx = \ln|x| + C \qquad \int \sin x\,dx = -\cos x + C$$

$$\int \cos x\,dx = \sin x + C$$

$$\int \frac{1}{\cos^2 x}\,dx = \tan x + C \qquad \int e^x\,dx = e^x + C$$

HL 5.12 Area to $x$- or $y$-axis $A = \int_a^b |y|\,dx \quad \text{or} \quad A = \int_a^b |x|\,dy$

HL Volume of revolution $V = \int_a^b \pi y^2\,dx \quad \text{or} \quad V = \int_a^b \pi x^2\,dy$

HL 5.13 Acceleration $a = \frac{dv}{dt} = \frac{d^2s}{dt^2} = v\frac{dv}{ds}$

HL Distance travelled $\text{distance} = \int_{t_1}^{t_2} |v(t)|\,dt$

HL Displacement $\text{displacement} = \int_{t_1}^{t_2} v(t)\,dt$

HL 5.16 Euler's method

$$y_{n+1} = y_n + h \times f(x_n, y_n); \quad x_{n+1} = x_n + h \text{ ($h$ constant step length)}$$

HL Euler, coupled systems

$$x_{n+1} = x_n + h \times f_1(x_n, y_n, t_n); \quad y_{n+1} = y_n + h \times f_2(x_n, y_n, t_n)$$

$$t_{n+1} = t_n + h \text{ ($h$ constant step length)}$$

HL 5.17 Coupled linear DEs: exact solution $\boldsymbol{x} = Ae^{\lambda_1 t}\boldsymbol{p}_1 + Be^{\lambda_2 t}\boldsymbol{p}_2$