IB Maths: Applications and Interpretation

HL formula booklet

Every formula you get in the exam, SL and HL content, in syllabus order

pietromeloni.com

Prior learning

	Area of a parallelogram	$A = bh$ (b base, h height)
	Area of a triangle	$A = \frac{1}{2}(bh)$ (b base, h height)
	Area of a trapezoid	$A = \frac{1}{2}(a+b)h$ (a, b parallel sides, h height)
	Area of a circle	$A = \pi r^2$ (r radius)
	Circumference of a circle	$C = 2\pi r$ (r radius)
	Volume of a cuboid	$V = lwh$ (l length, w width, h height)
	Volume of a cylinder	$V = \pi r^2 h$ (r radius, h height)
	Volume of a prism	$V = Ah$ (A area of cross-section, h height)
	Curved surface area of a cylinder	$A = 2\pi rh$ (r radius, h height)
	Distance between (x_1, y_1) and (x_2, y_2)	$d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$
	Midpoint of the segment from (x_1, y_1) to (x_2, y_2)	$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$
HL	Solutions of a quadratic equation	$ax^2 + bx + c = 0 \Rightarrow x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}, \quad a \neq 0$

Topic 1: Number and algebra

1.2	Arithmetic sequence: nth term	$u_n = u_1 + (n-1)d$
	Arithmetic sequence: sum of n terms	$S_n = \frac{n}{2}(2u_1 + (n-1)d); \quad S_n = \frac{n}{2}(u_1 + u_n)$
1.3	Geometric sequence: nth term	$u_n = u_1 r^{n-1}$
	Finite geometric sequence: sum of n terms	$S_n = \frac{u_1(r^n - 1)}{r - 1} = \frac{u_1(1 - r^n)}{1 - r}, \quad r \neq 1$
1.4	Compound interest	$FV = PV \times \left(1 + \frac{r}{100k}\right)^{kn}$ FV future value, PV present value, n number of years, k compounding periods per year, r% nominal annual rate of interest
1.5	Exponents and logarithms	$a^x = b \iff x = \log_a b, \quad a > 0,\ b > 0,\ a \neq 1$
1.6	Percentage error	$\varepsilon = \left\lvert \frac{v_A - v_E}{v_E} \right\rvert \times 100\%$ v_E exact value, v_A approximate value of v
1.9 HL	Laws of logarithms	$\log_a xy = \log_a x + \log_a y \qquad \log_a \frac{x}{y} = \log_a x - \log_a y$ $\log_a x^m = m \log_a x \qquad$ for $a, x, y > 0$
1.11 HL	Infinite geometric sequence: sum	$S_\infty = \frac{u_1}{1 - r}, \quad \lvert r \rvert < 1$
1.12 HL	Complex numbers	$z = a + b\mathrm{i}$
HL	Discriminant	$\Delta = b^2 - 4ac$
1.13 HL	Modulus-argument (polar) and exponential (Euler) form	$z = r(\cos\theta + \mathrm{i}\sin\theta) = r\mathrm{e}^{\mathrm{i}\theta} = r\,\mathrm{cis}\,\theta$

1.14 HL	Determinant of a 2×2 matrix	$\boldsymbol{A} = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \Rightarrow \det \boldsymbol{A} = \lvert\boldsymbol{A}\rvert = ad - bc$
HL	Inverse of a 2×2 matrix	$\boldsymbol{A} = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \Rightarrow \boldsymbol{A}^{-1} = \frac{1}{\det \boldsymbol{A}} \begin{pmatrix} d & -b \\ -c & a \end{pmatrix}, \quad ad \neq bc$
1.15 HL	Power of a diagonalisable matrix	$\boldsymbol{M}^n = \boldsymbol{P}\boldsymbol{D}^n\boldsymbol{P}^{-1}$ $\boldsymbol{P}$ matrix of eigenvectors, $\boldsymbol{D}$ diagonal matrix of eigenvalues

Topic 2: Functions

2.1	Equations of a straight line	$y = mx + c; \quad ax + by + d = 0; \quad y - y_1 = m(x - x_1)$
	Gradient formula	$m = \frac{y_2 - y_1}{x_2 - x_1}$
2.5	Axis of symmetry of the graph of a quadratic function	$f(x) = ax^2 + bx + c \Rightarrow$ axis of symmetry $x = -\frac{b}{2a}$
2.9 HL	Logistic function	$f(x) = \frac{L}{1 + Ce^{-kx}}, \quad L, k, C > 0$

Topic 3: Geometry and trigonometry

3.1	Distance between (x_1, y_1, z_1) and (x_2, y_2, z_2)	$d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2}$
	Midpoint of the segment from (x_1, y_1, z_1) to (x_2, y_2, z_2)	$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}, \frac{z_1 + z_2}{2}\right)$
	Volume of a right pyramid	$V = \frac{1}{3}Ah$ (A area of the base, h height)
	Volume of a right cone	$V = \frac{1}{3}\pi r^2 h$ (r radius, h height)
	Curved surface area of a cone	$A = \pi r l$ (r radius, l slant height)
	Volume of a sphere	$V = \frac{4}{3}\pi r^3$ (r radius)
	Surface area of a sphere	$A = 4\pi r^2$ (r radius)
3.2	Sine rule	$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$
	Cosine rule	$c^2 = a^2 + b^2 - 2ab\cos C; \quad \cos C = \frac{a^2 + b^2 - c^2}{2ab}$
	Area of a triangle	$A = \frac{1}{2}ab\sin C$
3.4	Length of an arc (degrees)	$l = \frac{\theta}{360} \times 2\pi r$ (θ angle in degrees, r radius)
	Area of a sector (degrees)	$A = \frac{\theta}{360} \times \pi r^2$ (θ angle in degrees, r radius)
3.7 HL	Length of an arc (radians)	$l = r\theta$ (r radius, θ angle in radians)
HL	Area of a sector (radians)	$A = \frac{1}{2}r^2\theta$ (r radius, θ angle in radians)
3.8 HL	Identities	$\cos^2\theta + \sin^2\theta = 1 \qquad \tan\theta = \frac{\sin\theta}{\cos\theta}$
3.9 HL	Transformation matrices	$\begin{pmatrix} \cos 2\theta & \sin 2\theta \\ \sin 2\theta & -\cos 2\theta \end{pmatrix}$ reflection in the line $y = (\tan\theta)x$ $\begin{pmatrix} k & 0 \\ 0 & k \end{pmatrix}$ enlargement, scale factor k, centre $(0, 0)$ $\begin{pmatrix} k & 0 \\ 0 & 1 \end{pmatrix}$ horizontal stretch (parallel to the x-axis), scale factor k $\begin{pmatrix} 1 & 0 \\ 0 & k \end{pmatrix}$ vertical stretch (parallel to the y-axis), scale factor k $\begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$ anticlockwise rotation by θ about the origin ($\theta > 0$) $\begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$ clockwise rotation by θ about the origin ($\theta > 0$)

3.10 HL	Magnitude of a vector	$\lvert\boldsymbol{v}\rvert = \sqrt{v_1^2 + v_2^2 + v_3^2}, \quad \text{where } \boldsymbol{v} = \begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix}$
3.11 HL	Vector equation of a line	$\boldsymbol{r} = \boldsymbol{a} + \lambda \boldsymbol{b}$
HL	Parametric form of the equation of a line	$x = x_0 + \lambda l, \quad y = y_0 + \lambda m, \quad z = z_0 + \lambda n$
3.13 HL	Scalar product	$\boldsymbol{v} \cdot \boldsymbol{w} = v_1 w_1 + v_2 w_2 + v_3 w_3, \quad \text{where } \boldsymbol{v} = \begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix}, \boldsymbol{w} = \begin{pmatrix} w_1 \\ w_2 \\ w_3 \end{pmatrix}$ $\boldsymbol{v} \cdot \boldsymbol{w} = \lvert\boldsymbol{v}\rvert \lvert\boldsymbol{w}\rvert \cos\theta$ (θ angle between $\boldsymbol{v}$ and $\boldsymbol{w}$)
HL	Angle between two vectors	$\cos\theta = \dfrac{v_1 w_1 + v_2 w_2 + v_3 w_3}{\lvert\boldsymbol{v}\rvert \lvert\boldsymbol{w}\rvert}$
HL	Vector product	$\boldsymbol{v} \times \boldsymbol{w} = \begin{pmatrix} v_2 w_3 - v_3 w_2 \\ v_3 w_1 - v_1 w_3 \\ v_1 w_2 - v_2 w_1 \end{pmatrix}, \quad \text{where } \boldsymbol{v} = \begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix}, \boldsymbol{w} = \begin{pmatrix} w_1 \\ w_2 \\ w_3 \end{pmatrix}$ $\lvert\boldsymbol{v} \times \boldsymbol{w}\rvert = \lvert\boldsymbol{v}\rvert \lvert\boldsymbol{w}\rvert \sin\theta$ (θ angle between $\boldsymbol{v}$ and $\boldsymbol{w}$)
HL	Area of a parallelogram	$A = \lvert\boldsymbol{v} \times \boldsymbol{w}\rvert$ ($\boldsymbol{v}$, $\boldsymbol{w}$ two adjacent sides)

Topic 4: Statistics and probability

4.2	Interquartile range	$\text{IQR} = Q_3 - Q_1$
4.3	Mean $\bar{x}$ of a set of data	$\bar{x} = \dfrac{\sum_{i=1}^{k} f_i x_i}{n}, \quad \text{where } n = \sum_{i=1}^{k} f_i$
4.5	Probability of an event A	$\text{P}(A) = \dfrac{n(A)}{n(U)}$
	Complementary events	$\text{P}(A) + \text{P}(A') = 1$
4.6	Combined events	$\text{P}(A \cup B) = \text{P}(A) + \text{P}(B) - \text{P}(A \cap B)$
	Mutually exclusive events	$\text{P}(A \cup B) = \text{P}(A) + \text{P}(B)$
	Conditional probability	$\text{P}(A \mid B) = \dfrac{\text{P}(A \cap B)}{\text{P}(B)}$
	Independent events	$\text{P}(A \cap B) = \text{P}(A)\,\text{P}(B)$
4.7	Expected value of a discrete random variable X	$\text{E}(X) = \sum x\,\text{P}(X = x)$
4.8	Binomial distribution $X \sim \text{B}(n, p)$	Mean $\quad \text{E}(X) = np$ Variance $\quad \text{Var}(X) = np(1-p)$
4.14 HL	Linear transformation of a single random variable	$\text{E}(aX + b) = a\text{E}(X) + b$ $\text{Var}(aX + b) = a^2\,\text{Var}(X)$
HL	Linear combinations of n independent random variables $X_1, X_2, \ldots, X_n$	$\text{E}(a_1X_1 \pm a_2X_2 \pm \cdots \pm a_nX_n) = a_1\text{E}(X_1) \pm a_2\text{E}(X_2) \pm \cdots \pm a_n\text{E}(X_n)$ $\text{Var}(a_1X_1 \pm a_2X_2 \pm \cdots \pm a_nX_n) = a_1^2\,\text{Var}(X_1) + a_2^2\,\text{Var}(X_2) + \cdots + a_n^2\,\text{Var}(X_n)$
HL	Sample statistics: unbiased estimate s_{n-1}^2 of the population variance	$s_{n-1}^2 = \dfrac{n}{n-1} s_n^2$
4.17 HL	Poisson distribution $X \sim \text{Po}(m)$	Mean $\quad \text{E}(X) = m$ Variance $\quad \text{Var}(X) = m$
4.19 HL	Transition matrices	$\boldsymbol{T}^n \boldsymbol{s}_0 = \boldsymbol{s}_n$ ($\boldsymbol{s}_0$ initial state)

Topic 5: Calculus

5.3	Derivative of x^n	$f(x) = x^n \Rightarrow f'(x) = nx^{n-1}$
5.5	Integral of x^n	$\int x^n \,\mathrm{d}x = \frac{x^{n+1}}{n+1} + C, \quad n \neq -1$
	Area between a curve $y = f(x)$ and the x-axis, where $f(x) > 0$	$A = \int_a^b y \,\mathrm{d}x$
5.8	The trapezoidal rule	$\int_a^b y \,\mathrm{d}x \approx \frac{1}{2}h\big((y_0 + y_n) + 2(y_1 + y_2 + \cdots + y_{n-1})\big), \quad \text{where } h = \frac{b-a}{n}$
5.9 HL	Derivative of $\sin x$	$f(x) = \sin x \Rightarrow f'(x) = \cos x$
HL	Derivative of $\cos x$	$f(x) = \cos x \Rightarrow f'(x) = -\sin x$
HL	Derivative of $\tan x$	$f(x) = \tan x \Rightarrow f'(x) = \frac{1}{\cos^2 x}$
HL	Derivative of e^x	$f(x) = \mathrm{e}^x \Rightarrow f'(x) = \mathrm{e}^x$
HL	Derivative of $\ln x$	$f(x) = \ln x \Rightarrow f'(x) = \frac{1}{x}$
HL	Chain rule	$y = g(u), \text{ where } u = f(x) \Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x} = \frac{\mathrm{d}y}{\mathrm{d}u} \times \frac{\mathrm{d}u}{\mathrm{d}x}$
HL	Product rule	$y = uv \Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x} = u\frac{\mathrm{d}v}{\mathrm{d}x} + v\frac{\mathrm{d}u}{\mathrm{d}x}$
HL	Quotient rule	$y = \frac{u}{v} \Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x} = \frac{v\frac{\mathrm{d}u}{\mathrm{d}x} - u\frac{\mathrm{d}v}{\mathrm{d}x}}{v^2}$
5.11 HL	Standard integrals	$\int \frac{1}{x} \,\mathrm{d}x = \ln\lvert x\rvert + C \quad \int \sin x \,\mathrm{d}x = -\cos x + C \quad \int \cos x \,\mathrm{d}x = \sin x + C$ $\int \frac{1}{\cos^2 x} \,\mathrm{d}x = \tan x + C \quad \int \mathrm{e}^x \,\mathrm{d}x = \mathrm{e}^x + C$
5.12 HL	Area between a curve and the x- or y-axis	$A = \int_a^b \lvert y\rvert \,\mathrm{d}x \quad \text{or} \quad A = \int_a^b \lvert x\rvert \,\mathrm{d}y$
HL	Volume of revolution about the x- or y-axis	$V = \int_a^b \pi y^2 \,\mathrm{d}x \quad \text{or} \quad V = \int_a^b \pi x^2 \,\mathrm{d}y$
5.13 HL	Acceleration	$a = \frac{\mathrm{d}v}{\mathrm{d}t} = \frac{\mathrm{d}^2 s}{\mathrm{d}t^2} = v\frac{\mathrm{d}v}{\mathrm{d}s}$
HL	Distance travelled from t_1 to t_2	$\text{distance} = \int_{t_1}^{t_2} \lvert v(t)\rvert \,\mathrm{d}t$
HL	Displacement from t_1 to t_2	$\text{displacement} = \int_{t_1}^{t_2} v(t) \,\mathrm{d}t$
5.16 HL	Euler's method	$y_{n+1} = y_n + h \times f(x_n, y_n); \quad x_{n+1} = x_n + h$ (h constant step length)
HL	Euler's method for coupled systems	$x_{n+1} = x_n + h \times f_1(x_n, y_n, t_n); \quad y_{n+1} = y_n + h \times f_2(x_n, y_n, t_n)$ $t_{n+1} = t_n + h$ (h constant step length)
5.17 HL	Exact solution for coupled linear differential equations	$\boldsymbol{x} = A\mathrm{e}^{\lambda_1 t}\boldsymbol{p}_1 + B\mathrm{e}^{\lambda_2 t}\boldsymbol{p}_2$

Re-typeset from the official IB Mathematics: applications and interpretation formula booklet (first examinations 2021).
Not an official IB publication: in the exam you receive the official booklet.