

Prior learning

Area of a parallelogram	$A = bh$ (b base, h height)
Area of a triangle	$A = \frac{1}{2}(bh)$ (b base, h height)
Area of a trapezoid	$A = \frac{1}{2}(a + b)h$ (a, b parallel sides, h height)
Area of a circle	$A = \pi r^2$ (r radius)
Circumference of a circle	$C = 2\pi r$ (r radius)
Volume of a cuboid	$V = lwh$ (l length, w width, h height)
Volume of a cylinder	$V = \pi r^2 h$ (r radius, h height)
Volume of a prism	$V = Ah$ (A area of cross-section, h height)
Area of the curved surface of a cylinder	$A = 2\pi rh$ (r radius, h height)
Distance between (x_1, y_1) and (x_2, y_2)	$d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$
Midpoint of the segment from (x_1, y_1) to (x_2, y_2)	$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$

Topic 1: Number and algebra

1.2	The n th term of an arithmetic sequence	$u_n = u_1 + (n - 1)d$
	The sum of n terms of an arithmetic sequence	$S_n = \frac{n}{2}(2u_1 + (n - 1)d); \quad S_n = \frac{n}{2}(u_1 + u_n)$
1.3	The n th term of a geometric sequence	$u_n = u_1 r^{n-1}$
	The sum of n terms of a finite geometric sequence	$S_n = \frac{u_1(r^n - 1)}{r - 1} = \frac{u_1(1 - r^n)}{1 - r}, \quad r \neq 1$
1.4	Compound interest	$FV = PV \times \left(1 + \frac{r}{100k}\right)^{kn}$ FV future value, PV present value, n number of years, k compounding periods per year, $r\%$ nominal annual rate of interest
1.5	Exponents and logarithms	$a^x = b \iff x = \log_a b, \quad a > 0, b > 0, a \neq 1$
1.7	Exponents and logarithms	$\log_a xy = \log_a x + \log_a y \quad \log_a \frac{x}{y} = \log_a x - \log_a y$ $\log_a x^m = m \log_a x \quad \log_a x = \frac{\log_b x}{\log_b a}$
1.8	The sum of an infinite geometric sequence	$S_\infty = \frac{u_1}{1 - r}, \quad r < 1$
1.9	Binomial theorem	$(a + b)^n = a^n + {}^nC_1 a^{n-1} b + \dots + {}^nC_r a^{n-r} b^r + \dots + b^n$ ${}^nC_r = \frac{n!}{r!(n-r)!}$
1.10 HL	Combinations	${}^nC_r = \frac{n!}{r!(n-r)!}$
HL	Permutations	${}^nP_r = \frac{n!}{(n-r)!}$
1.12 HL	Complex numbers	$z = a + bi$

1.13 HL	Modulus-argument (polar) and exponential (Euler) form	$z = r(\cos \theta + i \sin \theta) = re^{i\theta} = r \operatorname{cis} \theta$
1.14 HL	De Moivre's theorem	$[r(\cos \theta + i \sin \theta)]^n = r^n(\cos n\theta + i \sin n\theta) = r^n e^{in\theta} = r^n \operatorname{cis} n\theta$

Topic 2: Functions

2.1	Equations of a straight line	$y = mx + c; \quad ax + by + d = 0; \quad y - y_1 = m(x - x_1)$
	Gradient formula	$m = \frac{y_2 - y_1}{x_2 - x_1}$
2.6	Axis of symmetry of the graph of a quadratic	$f(x) = ax^2 + bx + c \Rightarrow \text{axis of symmetry } x = -\frac{b}{2a}$
2.7	Solutions of a quadratic equation	$ax^2 + bx + c = 0 \Rightarrow x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}, \quad a \neq 0$
	Discriminant	$\Delta = b^2 - 4ac$
2.9	Exponential and logarithmic functions	$a^x = e^{x \ln a}; \quad \log_a a^x = x = a^{\log_a x}, \quad a, x > 0, a \neq 1$
2.12 HL	Sum and product of the roots of $\sum_{r=0}^n a_r x^r = 0$	Sum = $\frac{-a_{n-1}}{a_n}$; product = $\frac{(-1)^n a_0}{a_n}$

Topic 3: Geometry and trigonometry

3.1	Distance between (x_1, y_1, z_1) and (x_2, y_2, z_2)	$d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2}$
	Midpoint of the segment from (x_1, y_1, z_1) to (x_2, y_2, z_2)	$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}, \frac{z_1 + z_2}{2}\right)$
	Volume of a right pyramid	$V = \frac{1}{3}Ah$ (A area of the base, h height)
	Volume of a right cone	$V = \frac{1}{3}\pi r^2 h$ (r radius, h height)
	Area of the curved surface of a cone	$A = \pi r l$ (r radius, l slant height)
	Volume of a sphere	$V = \frac{4}{3}\pi r^3$ (r radius)
	Surface area of a sphere	$A = 4\pi r^2$ (r radius)
3.2	Sine rule	$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$
	Cosine rule	$c^2 = a^2 + b^2 - 2ab \cos C; \quad \cos C = \frac{a^2 + b^2 - c^2}{2ab}$
	Area of a triangle	$A = \frac{1}{2}ab \sin C$
3.4	Length of an arc	$l = r\theta$ (r radius, θ angle in radians)
	Area of a sector	$A = \frac{1}{2}r^2\theta$ (r radius, θ angle in radians)
3.5	Identity for $\tan \theta$	$\tan \theta = \frac{\sin \theta}{\cos \theta}$
3.6	Pythagorean identity	$\cos^2 \theta + \sin^2 \theta = 1$
	Double angle identities	$\sin 2\theta = 2 \sin \theta \cos \theta$ $\cos 2\theta = \cos^2 \theta - \sin^2 \theta = 2 \cos^2 \theta - 1 = 1 - 2 \sin^2 \theta$
3.9 HL	Reciprocal trigonometric identities	$\sec \theta = \frac{1}{\cos \theta} \quad \operatorname{cosec} \theta = \frac{1}{\sin \theta}$
HL	Pythagorean identities	$1 + \tan^2 \theta = \sec^2 \theta \quad 1 + \cot^2 \theta = \operatorname{cosec}^2 \theta$

3.10 HL	Compound angle identities	$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$ $\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$ $\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$
HL	Double angle identity for tan	$\tan 2\theta = \frac{2 \tan \theta}{1 - \tan^2 \theta}$
3.12 HL	Magnitude of a vector	$ \mathbf{v} = \sqrt{v_1^2 + v_2^2 + v_3^2}$ where $\mathbf{v} = \begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix}$
3.13 HL	Scalar product	$\mathbf{v} \cdot \mathbf{w} = v_1 w_1 + v_2 w_2 + v_3 w_3$ where $\mathbf{v} = \begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix}, \mathbf{w} = \begin{pmatrix} w_1 \\ w_2 \\ w_3 \end{pmatrix}$ $\mathbf{v} \cdot \mathbf{w} = \mathbf{v} \mathbf{w} \cos \theta$ (θ angle between $\mathbf{v}$ and $\mathbf{w}$)
HL	Angle between two vectors	$\cos \theta = \frac{v_1 w_1 + v_2 w_2 + v_3 w_3}{ \mathbf{v} \mathbf{w} }$
3.14 HL	Vector equation of a line	$\mathbf{r} = \mathbf{a} + \lambda \mathbf{b}$
HL	Parametric form of the equation of a line	$x = x_0 + \lambda l, \quad y = y_0 + \lambda m, \quad z = z_0 + \lambda n$
HL	Cartesian equations of a line	$\frac{x - x_0}{l} = \frac{y - y_0}{m} = \frac{z - z_0}{n}$
3.16 HL	Vector product	$\mathbf{v} \times \mathbf{w} = \begin{pmatrix} v_2 w_3 - v_3 w_2 \\ v_3 w_1 - v_1 w_3 \\ v_1 w_2 - v_2 w_1 \end{pmatrix}$ where $\mathbf{v} = \begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix}, \mathbf{w} = \begin{pmatrix} w_1 \\ w_2 \\ w_3 \end{pmatrix}$ $ \mathbf{v} \times \mathbf{w} = \mathbf{v} \mathbf{w} \sin \theta$ (θ angle between $\mathbf{v}$ and $\mathbf{w}$)
HL	Area of a parallelogram	$A = \mathbf{v} \times \mathbf{w} $ ($\mathbf{v}, \mathbf{w}$ two adjacent sides)
3.17 HL	Vector equation of a plane	$\mathbf{r} = \mathbf{a} + \lambda \mathbf{b} + \mu \mathbf{c}$
HL	Equation of a plane (using the normal vector)	$\mathbf{r} \cdot \mathbf{n} = \mathbf{a} \cdot \mathbf{n}$
HL	Cartesian equation of a plane	$ax + by + cz = d$

Topic 4: Statistics and probability

4.2	Interquartile range	$\text{IQR} = Q_3 - Q_1$
4.3	Mean $\bar{x}$ of a set of data	$\bar{x} = \frac{\sum_{i=1}^k f_i x_i}{n}, \quad \text{where } n = \sum_{i=1}^k f_i$
4.5	Probability of an event A	$P(A) = \frac{n(A)}{n(U)}$
	Complementary events	$P(A) + P(A') = 1$
4.6	Combined events	$P(A \cup B) = P(A) + P(B) - P(A \cap B)$
	Mutually exclusive events	$P(A \cup B) = P(A) + P(B)$
	Conditional probability	$P(A B) = \frac{P(A \cap B)}{P(B)}$
	Independent events	$P(A \cap B) = P(A)P(B)$
4.7	Expected value of a discrete random variable X	$E(X) = \sum xP(X = x)$
4.8	Binomial distribution $X \sim B(n, p)$: mean and variance	$E(X) = np \quad \text{Var}(X) = np(1 - p)$
4.12	Standardized normal variable	$z = \frac{x - \mu}{\sigma}$

4.13 HL	Bayes' theorem	$P(B A) = \frac{P(B)P(A B)}{P(B)P(A B) + P(B')P(A B')}$ $P(B_i A) = \frac{P(B_i)P(A B_i)}{P(B_1)P(A B_1) + P(B_2)P(A B_2) + P(B_3)P(A B_3)}$
4.14 HL	Variance σ^2	$\sigma^2 = \frac{\sum_{i=1}^k f_i(x_i - \mu)^2}{n} = \frac{\sum_{i=1}^k f_i x_i^2}{n} - \mu^2$
HL	Standard deviation σ	$\sigma = \sqrt{\frac{\sum_{i=1}^k f_i(x_i - \mu)^2}{n}}$
HL	Linear transformation of a single random variable	$E(aX + b) = aE(X) + b \quad \text{Var}(aX + b) = a^2 \text{Var}(X)$
HL	Expected value of a continuous random variable X	$E(X) = \mu = \int_{-\infty}^{\infty} x f(x) dx$
HL	Variance	$\text{Var}(X) = E(X - \mu)^2 = E(X^2) - [E(X)]^2$
HL	Variance of a discrete random variable X	$\text{Var}(X) = \sum (x - \mu)^2 P(X = x) = \sum x^2 P(X = x) - \mu^2$
HL	Variance of a continuous random variable X	$\text{Var}(X) = \int_{-\infty}^{\infty} (x - \mu)^2 f(x) dx = \int_{-\infty}^{\infty} x^2 f(x) dx - \mu^2$

Topic 5: Calculus

5.3	Derivative of x^n	$f(x) = x^n \Rightarrow f'(x) = nx^{n-1}$
5.5	Integral of x^n	$\int x^n dx = \frac{x^{n+1}}{n+1} + C, \quad n \neq -1$
	Area between the curve $y = f(x)$ and the x -axis, where $f(x) > 0$	$A = \int_a^b y dx$
5.6	Derivative of $\sin x$	$f(x) = \sin x \Rightarrow f'(x) = \cos x$
	Derivative of $\cos x$	$f(x) = \cos x \Rightarrow f'(x) = -\sin x$
	Derivative of e^x	$f(x) = e^x \Rightarrow f'(x) = e^x$
	Derivative of $\ln x$	$f(x) = \ln x \Rightarrow f'(x) = \frac{1}{x}$
	Chain rule	$y = g(u), \text{ where } u = f(x) \Rightarrow \frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$
	Product rule	$y = uv \Rightarrow \frac{dy}{dx} = u \frac{dv}{dx} + v \frac{du}{dx}$
	Quotient rule	$y = \frac{u}{v} \Rightarrow \frac{dy}{dx} = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2}$
5.9	Acceleration	$a = \frac{dv}{dt} = \frac{d^2s}{dt^2}$
	Distance travelled from t_1 to t_2	$\text{distance} = \int_{t_1}^{t_2} v(t) dt$
	Displacement from t_1 to t_2	$\text{displacement} = \int_{t_1}^{t_2} v(t) dt$
5.10	Standard integrals	$\int \frac{1}{x} dx = \ln x + C \quad \int \sin x dx = -\cos x + C$ $\int \cos x dx = \sin x + C \quad \int e^x dx = e^x + C$
5.11	Area of the region enclosed by a curve and the x -axis	$A = \int_a^b y dx$

5.12 HL	Derivative of $f(x)$ from first principles	$y = f(x) \Rightarrow \frac{dy}{dx} = f'(x) = \lim_{h \rightarrow 0} \left(\frac{f(x+h) - f(x)}{h} \right)$
5.15 HL	Standard derivatives: $\tan x$, $\sec x$	$f(x) = \tan x \Rightarrow f'(x) = \sec^2 x$ $f(x) = \sec x \Rightarrow f'(x) = \sec x \tan x$
HL	Standard derivatives: $\operatorname{cosec} x$, $\cot x$	$f(x) = \operatorname{cosec} x \Rightarrow f'(x) = -\operatorname{cosec} x \cot x$ $f(x) = \cot x \Rightarrow f'(x) = -\operatorname{cosec}^2 x$
HL	Standard derivatives: a^x , $\log_a x$	$f(x) = a^x \Rightarrow f'(x) = a^x (\ln a)$ $f(x) = \log_a x \Rightarrow f'(x) = \frac{1}{x \ln a}$
HL	Standard derivatives: $\arcsin x$, $\arccos x$, $\arctan x$	$f(x) = \arcsin x \Rightarrow f'(x) = \frac{1}{\sqrt{1-x^2}}$ $f(x) = \arccos x \Rightarrow f'(x) = -\frac{1}{\sqrt{1-x^2}}$ $f(x) = \arctan x \Rightarrow f'(x) = \frac{1}{1+x^2}$
HL	Standard integrals	$\int a^x dx = \frac{1}{\ln a} a^x + C$ $\int \frac{1}{a^2 + x^2} dx = \frac{1}{a} \arctan\left(\frac{x}{a}\right) + C$ $\int \frac{1}{\sqrt{a^2 - x^2}} dx = \arcsin\left(\frac{x}{a}\right) + C, \quad x < a$
5.16 HL	Integration by parts	$\int u \frac{dv}{dx} dx = uv - \int v \frac{du}{dx} dx \quad \text{or} \quad \int u dv = uv - \int v du$
5.17 HL	Area of the region enclosed by a curve and the y -axis	$A = \int_a^b x dy$
HL	Volume of revolution about the x - or y -axis	$V = \int_a^b \pi y^2 dx \quad \text{or} \quad V = \int_a^b \pi x^2 dy$
5.18 HL	Euler's method	$y_{n+1} = y_n + h \times f(x_n, y_n); \quad x_{n+1} = x_n + h$ (h constant step length)
HL	Integrating factor for $y' + P(x)y = Q(x)$	$e^{\int P(x) dx}$
5.19 HL	Maclaurin series	$f(x) = f(0) + x f'(0) + \frac{x^2}{2!} f''(0) + \dots$
HL	Maclaurin series for special functions	$e^x = 1 + x + \frac{x^2}{2!} + \dots \quad \ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \dots$ $\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \dots \quad \cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \dots$ $\arctan x = x - \frac{x^3}{3} + \frac{x^5}{5} - \dots$