Predicted paper · May 2027

IB Mathematics: applications and interpretation

Higher level · Paper 1

2 hours

Instructions

- 16 short questions, 110 marks. Answer in the spaces provided.
- Total: 110 marks. Answer all questions.
- A graphic display calculator is required for this paper.
- Full marks are not necessarily awarded for a correct answer with no working: show your method.
- You need a clean copy of the mathematics: applications and interpretation formula booklet.
- Sit it in one go, timed. Then mark it with the mark scheme, honestly.

Name .....

Date .....

Score .....

Every question in this paper is new, written by Pietro Meloni to the shape of the papers set on the current syllabus. It is not IB material and it is not a leaked paper: it is a prediction of the topics and the style, for practice under timed conditions.

Mark scheme and the other papers in the set: scan the code

1. A vineyard collects rainwater in a cylindrical tank of radius 1.35 m and height 2.6 m.

(a) Calculate the volume of the tank in m^3. **[2]**

(b) Write your answer to part (a) in litres, in the form $a \times 10^k$ where $1 \leq a < 10$ and $k \in \mathbb{Z}$. ($1\ \text{m}^3 = 1000$ litres) **[1]**

(c) The owner of the vineyard estimates the volume of the tank to be $15\ \text{m}^3$. Calculate the percentage error in this estimate. **[2]**

(d) The height of the tank, 2.6 m, was measured correct to the nearest 10 cm. Write down the lower bound and the upper bound of the height. **[1]**

2. Marta borrows 18 000 EUR to buy a car. The bank offers her a loan to be repaid with equal monthly payments over 4 years, at a nominal annual interest rate of 6.9% compounded monthly.

(a) Find the amount of each monthly payment. **[3]**

(b) Find the total amount of interest Marta pays over the 4 years. **[1]**

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(c) A second bank offers the same loan over 4 years at a nominal annual interest rate of 5.4% compounded monthly, but charges an arrangement fee of 450 EUR which Marta would pay immediately. Determine which offer costs Marta less in total, and by how much. **[2]**

3. A fountain sends a jet of water from a nozzle into a pool. The height of the jet above the surface of the pool, h metres, at a horizontal distance of x metres from the nozzle is modelled by

$$h(x) = ax^2 + bx + 0.6, \qquad x \geq 0,$$

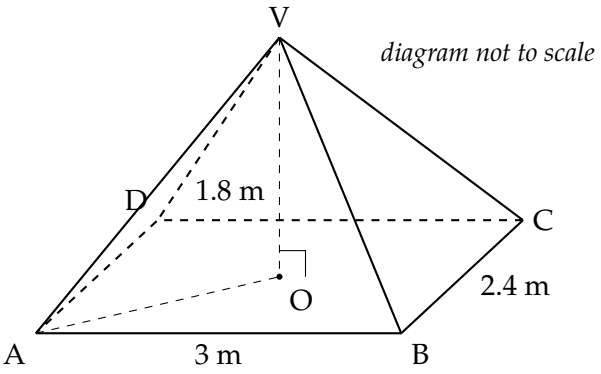
where a and b are constants. The jet passes through the points $(1, 1.7)$ and $(2, 2.0)$.

(a) Find the value of a and the value of b. **[3]**

(b) Find the maximum height of the jet above the surface of the pool. **[2]**

(c) The jet lands on the surface of the pool at a horizontal distance of d metres from the nozzle. Find the value of d and hence write down the domain of the model. **[2]**

4. A camping tent has the shape of a right pyramid VABCD with a rectangular base ABCD in which AB = 3 m and BC = 2.4 m. The vertex V is 1.8 m vertically above the centre O of the base.

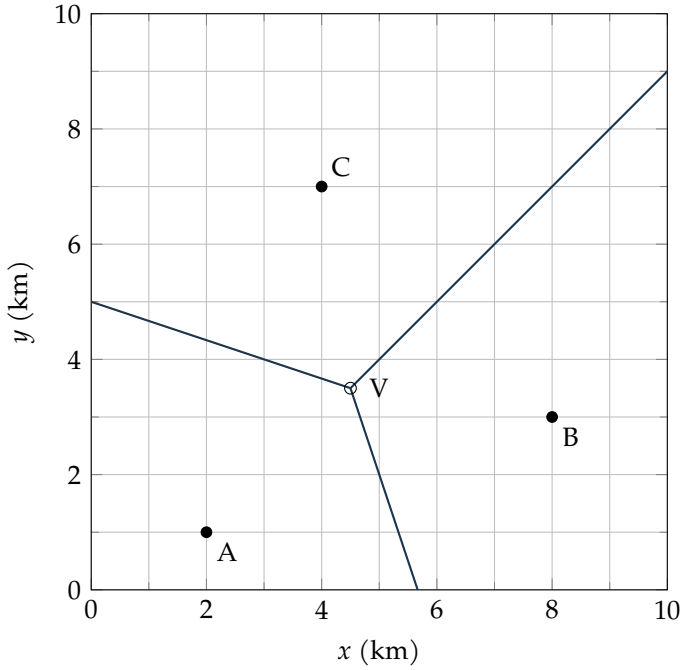


(**a**) Find the length of the sloping edge VA. **[2]**

(**b**) Find the angle that the edge VA makes with the base of the tent. **[2]**

(**c**) The four sloping faces of the tent are made of fabric. Find the total area of fabric needed. **[3]**

5. A town has three pharmacies, located at the points A(2, 1), B(8, 3) and C(4, 7) on a map in which the units are kilometres. The Voronoi diagram for the three sites is shown below. Residents go to the pharmacy nearest to their home.



(a) Find the equation of the perpendicular bisector of [AB]. Give your answer in the form $y = mx + c$. **[3]**

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(b) The edge between the cells of B and C lies on the line $y = x - 1$. Find the coordinates of the vertex V of the Voronoi diagram. **[2]**

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(c) A fourth pharmacy is to be opened at the point inside triangle ABC that is as far as possible from the nearest existing pharmacy. Write down the coordinates of this point and find its distance from the nearest existing pharmacy. **[2]**

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6. A patient takes a single 400 mg dose of a painkiller. The amount of the drug in the patient's blood, A mg, t hours after the dose is taken is modelled by

$$A(t) = 400\,\mathrm{e}^{-kt}, \qquad t \geq 0,$$

where k is a positive constant. After 3 hours there are 260 mg of the drug in the blood.

(a) Find the value of k. **[2]**

(b) Find the amount of the drug in the blood 8 hours after the dose is taken. **[2]**

(c) The painkiller is effective while there are at least 80 mg of the drug in the blood. Find the length of time for which the painkiller is effective. **[2]**

(d) State one limitation of this model. **[1]**

7. In an alternating current circuit, the voltage V, the current I and the impedance Z are represented by complex numbers that satisfy $V = IZ$. In a particular circuit the impedance is $Z = 12 + 9\mathrm{i}$ ohms and the current is $I = 5\,\mathrm{e}^{\mathrm{i}\frac{\pi}{6}}$ amperes.

(a) Find the modulus and the argument of Z. **[2]**

(b) Hence write V in the form $r\,\mathrm{e}^{\mathrm{i}\theta}$, where $r > 0$ and $0 \leq \theta < 2\pi$. **[2]**

(c) Write V in the form $a + b\mathrm{i}$, where $a, b \in \mathbb{R}$. **[2]**

(d) The phase difference between the voltage and the current is $\arg V - \arg I$. Write down the phase difference in this circuit. **[1]**

8. A ferry and a cargo ship move with constant velocities. At time t hours after 10:00 their positions, in kilometres relative to a port, are

$$\mathbf{r}_{\mathrm{F}} = \begin{pmatrix} 2 \\ 15 \end{pmatrix} + t \begin{pmatrix} 6 \\ -8 \end{pmatrix} \qquad \text{and} \qquad \mathbf{r}_{\mathrm{C}} = \begin{pmatrix} -10 \\ 3 \end{pmatrix} + t \begin{pmatrix} 9 \\ 1 \end{pmatrix}.$$

(a) Find the speed of the ferry. **[2]**

(b) Show that the distance between the two ships at time t is $\sqrt{90t^2 - 288t + 288}$ kilometres. **[2]**

(c) Find the minimum distance between the two ships and the time of day at which it occurs. **[2]**

(d) The radar of the cargo ship detects any vessel that is less than 10 km away. Find the length of time, in minutes, for which the ferry is detected by the radar. **[2]**

9. A vineyard has 12 rows of vines. At harvest, the mass of grapes picked from each row is recorded, in kilograms. The masses, in order, are

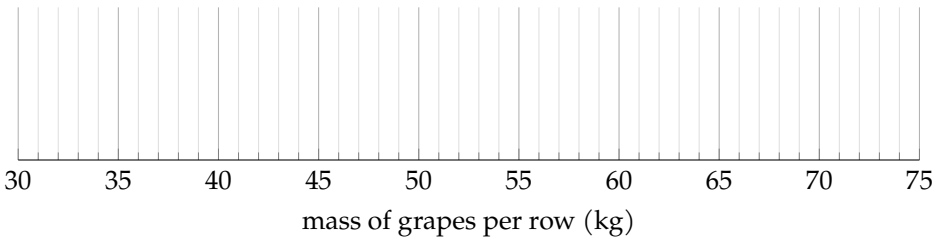
38 41 44 44 46 47 49 50 52 52 55 71

(a) Write down the median mass. **[1]**

(b) Find the interquartile range of the masses. **[2]**

(c) Show that the mass of 71 kg is an outlier. **[2]**

(d) On the axes below, draw a box-and-whisker diagram for the data. **[1]**



10. A cinema shows the same film on eight evenings, charging a different ticket price each evening. The table shows the ticket price, x euros, and the number of tickets sold, y, for each evening.

Price x (euros)	6	8	9	11	12	14	15	18
Tickets sold y	205	186	190	158	149	142	118	101

(a)

(i) Find Pearson's product-moment correlation coefficient, r, for these data. **[2]**

(ii) Describe the correlation between x and y. **[1]**

(b) Find the equation of the regression line of y on x. **[2]**

(c) Use your regression line to estimate the number of tickets sold on an evening when the price is 10 euros. **[1]**

(d) The manager uses the regression line to predict the number of tickets sold when the price is 30 euros. Give one reason why this prediction is not valid. **[1]**

11. During one night, 120 patients arrive at the emergency department of a hospital. Each patient either arrives by ambulance or walks in, and the triage nurse classifies each patient as urgent or non-urgent. The results are shown in the table.

	Urgent	Non-urgent	Total
Arrived by ambulance	28	14	42
Walked in	22	56	78
Total	50	70	120

One of these patients is chosen at random.

(a) Write down the probability that the patient was classified as urgent. **[1]**

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(b) Find the probability that the patient arrived by ambulance, given that the patient was classified as urgent. **[2]**

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(c) Determine, with a reason, whether the events "the patient arrived by ambulance" and "the patient was classified as urgent" are independent. **[3]**

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12. The times taken by the members of a swimming club to swim 100 metres are normally distributed with mean 72.5 seconds and standard deviation 4.8 seconds. One of these swims is chosen at random.

(a) Find the probability that the time taken is less than 65 seconds. **[2]**

(b) Find the probability that the time taken is between 70 seconds and 80 seconds. **[2]**

(c) The fastest 10 % of swims qualify for a regional competition. Find the slowest time, in seconds, that qualifies. **[2]**

(d) During one season, the club records 150 swims. Find the expected number of these swims with a time of less than 65 seconds. **[1]**

13. The organisers of a music festival ask 280 visitors which of the three stages they prefer: the main stage, the electronic stage or the acoustic stage. The results, by age group, are shown in the table.

	Main stage	Electronic stage	Acoustic stage
Under 25	36	42	22
25 to 40	39	32	29
Over 40	27	18	35

The organisers carry out a χ^2 test for independence at the 5 % significance level.

(a) State the null hypothesis and the alternative hypothesis. **[1]**

(**b**) Write down the number of degrees of freedom. **[1]**

(**c**) Find the expected number of visitors aged under 25 who prefer the main stage, assuming the null hypothesis is true. **[2]**

(**d**) Find the p-value for this test. **[2]**

(**e**) State the conclusion of the test, giving a reason for your answer. **[1]**

14. A section of a mountain bike trail is 15 metres long and crosses a small hill. The height of the trail above a fixed level, y metres, is modelled by

$$y = -0.01x^3 + 0.18x^2 - 0.6x + 30, \qquad 0 \le x \le 15,$$

where x is the horizontal distance, in metres, from the start of the section.

(**a**) Find $\dfrac{\mathrm{d}y}{\mathrm{d}x}$. **[2]**

(**b**) Find the equation of the tangent to the curve at the point where $x = 6$. Give your answer in the form $y = mx + c$. **[2]**

(c) Find the horizontal distance from the start of the section at which the trail is highest, and the height of the trail at that point. **[3]**

15. A glass vase is modelled by rotating the curve

$$y = 4 + 1.5\sin(0.3x), \qquad 0 \leq x \leq 20,$$

through 360° about the x-axis, where x and y are measured in centimetres and $0.3x$ is in radians. The vase is 20 cm tall.

(a) Find $\dfrac{\mathrm{d}y}{\mathrm{d}x}$. **[2]**

(b) Find the value of x at which the vase is widest. **[2]**

(c) Find the volume of the vase. **[3]**

16. A wind farm has 30 turbines. Faults that need a technician occur at random and independently, at a mean rate of 1.8 faults per week. The number of faults in a given period of time is modelled by a Poisson distribution.

(a) Find the probability that exactly two faults occur in a given week. **[2]**

(b) Find the probability that at least one fault occurs in a given period of two days. **[2]**

(c) A new maintenance scheme is introduced. In the eight weeks that follow, 7 faults occur. Test, at the 5 % significance level, whether the mean rate of faults has decreased. State your hypotheses, the p-value and your conclusion in context. **[4]**

Predicted paper · May 2027

IB Mathematics: applications and interpretation

Higher level · Paper 2

2 hours

Instructions

- 7 longer questions, 110 marks. Answer on lined paper.
- Total: 110 marks. Answer all questions.
- A graphic display calculator is required for this paper.
- Full marks are not necessarily awarded for a correct answer with no working: show your method.
- You need a clean copy of the mathematics: applications and interpretation formula booklet.
- Sit it in one go, timed. Then mark it with the mark scheme, honestly.

Name .....

Date .....

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Mark scheme and the other papers in the set: scan the code

1. Lucia makes and sells scented candles. Her monthly cost, in euros (EUR), of making x candles is modelled by

$$C(x) = 1800 + 4.5x.$$

To sell more candles she lowers the price: when she sells x candles in a month, each candle is sold for $(22 - 0.01x)$ EUR, for $0 \leq x \leq 2000$.

(a) Find the selling price of each candle when Lucia sells 500 candles in a month. **[1]**

(b) Show that Lucia's monthly profit, P EUR, is given by $P(x) = -0.01x^2 + 17.5x - 1800$. **[2]**

(c) Find the values of x for which Lucia breaks even. **[2]**

(d) Hence write down the least number of candles Lucia must sell in a month to make a profit. **[1]**

(e) Find the number of candles Lucia should sell in a month to maximise her profit, and find this maximum monthly profit. **[3]**

(f) At the end of the year Lucia invests 12 000 EUR in a savings account that pays a nominal annual interest rate of 4.2%, compounded quarterly.

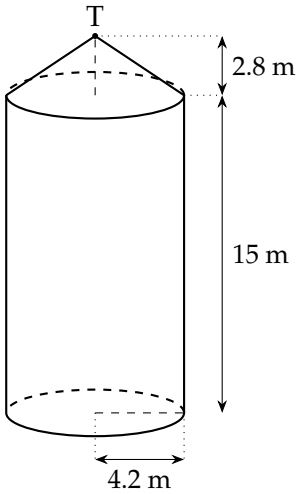
(i) Find the value of the investment after 5 years. **[2]**

(ii) The annual rate of inflation is expected to be 2.6% for the next 5 years. Find the real value of the investment after 5 years, in terms of today's money. **[2]**

(g) Lucia also opens an online shop. She models the number of candles sold online in the tth month after it opens by $O(t) = 150 \times 1.065^t$, and the number sold in her market stall in the same month by $S(t) = 720 - 12t$. Find the first month in which, according to these models, she sells more candles online than at the market stall. **[3]**

2. A grain silo stands on level ground. It is made of a cylinder of radius 4.2 m and height 15 m, with a roof in the shape of a right cone of radius 4.2 m and height 2.8 m. The apex of the roof is T.

diagram not to scale



(a) Find the volume of the silo. **[2]**

(b) Find the total area of the curved surface of the cylinder and of the curved surface of the roof. **[3]**

(c) The curved surfaces in part (b) are to be painted. One litre of paint covers 7.5 m^2, and the paint is sold only in 10-litre cans costing 64 EUR each. Find the cost of the paint needed. **[2]**

(d) A point P on the ground is 30 m from the nearest point of the base of the silo. Find the angle of elevation of T from P. **[2]**

On the farm, the silo S is 64 m from the farmhouse H and the barn B is 85 m from H. The angle $S\hat{H}B$ is 52°.

(e) Find the distance SB. **[2]**

(f) Find the angle $H\hat{B}S$. **[2]**

(g) On a plan of the farm, with distances in metres, H is at the origin, B is at (85, 0) and S is at (39.4, 50.4). A water trough W is to be placed on the straight track [HB] so that it is the same distance from S as from B. By considering the perpendicular bisector of [SB], or otherwise, find the coordinates of W. **[3]**

3. A researcher wants to know whether students who keep their phone in the bedroom at night sleep less than students who do not. A school has 960 students: 400 in Year 12 and 560 in Year 13. The researcher chooses a stratified sample of 24 students by year group.

(a) Find the number of Year 12 students in the sample. **[2]**

(b) State one reason why a stratified sample is preferable to choosing the first 24 students who arrive at school one morning. **[1]**

Each student in the sample reports in a questionnaire their mean number of hours of sleep per night over one week. The results, in hours, are shown in the table.

Phone in bedroom (13 students)	6.4	7.1	5.8	6.9	7.4	6.2	6.6	7.8	5.9	6.8	7.0	6.3	7.2
No phone (11 students)	7.5	6.9	8.1	7.2	6.6	7.9	7.4	8.3	7.0	7.6	6.8		

(c) Find the mean and the standard deviation of the sleep times for each of the two groups. **[3]**

(d) The researcher carries out a t-test, at the 5% significance level, to test whether the mean sleep time of students with a phone in the bedroom is less than the mean sleep time of students without one.

(i) State the null and alternative hypotheses. **[1]**

(ii) Find the p-value for this test. **[2]**

(iii) State the conclusion of the test, giving a reason. **[2]**

(e) Two of the 13 students in the sample with a phone in the bedroom are chosen at random. Find the probability that both of them sleep fewer than 7 hours per night. **[3]**

(f) The researcher concludes that keeping a phone in the bedroom makes students sleep less.

(i) State one assumption about the populations that is needed for the t-test to be valid. **[1]**

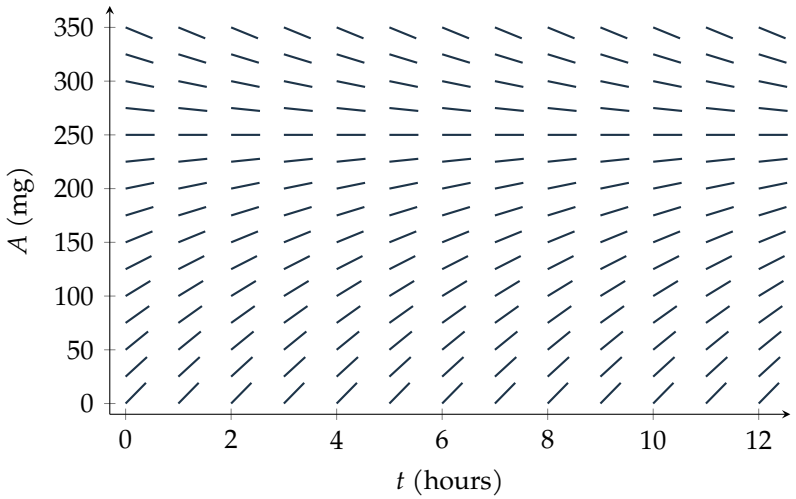
(ii) Give one reason why this conclusion may not be reliable. **[1]**

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4. A patient receives a drug through an intravenous drip. The drug enters the blood at a constant rate of 50 mg per hour, and the body removes the drug at a rate proportional to the amount present. Let A mg be the amount of the drug in the blood t hours after the drip is started. When $A = 100$, the amount of the drug is increasing at a rate of 30 mg per hour.

(a) Show that $\dfrac{\mathrm{d}A}{\mathrm{d}t} = 50 - 0.2A$. **[2]**

(b) The slope field of this differential equation is shown below.



(i) Write down the equilibrium solution of the differential equation. **[1]**

(ii) A second patient already has 300 mg of the drug in the blood when the same drip is started. Using the slope field, describe how the amount of the drug in this patient's blood changes over time. **[1]**

For the first patient, $A = 0$ when $t = 0$.

(c) Use Euler's method with a step length of 0.5 hours to estimate the amount of the drug in the blood after 1.5 hours. **[3]**

(d) By solving the differential equation, show that $A = 250\left(1 - \mathrm{e}^{-0.2t}\right)$. **[4]**

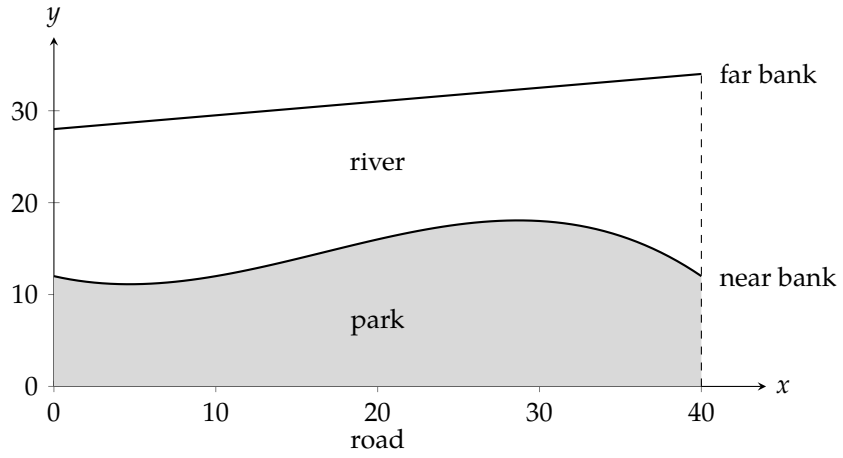
(e) Calculate the percentage error in the estimate found in part (c). **[2]**

(f) The drug is effective when there are at least 200 mg of it in the blood. Find the time it takes for the drug to become effective. **[2]**

5. A town is building a riverside park between a straight road and a river. On a plan, the edge of the road lies along the x-axis and the park occupies the region between the road, the near bank of the river and the lines $x = 0$ and $x = 40$. The near bank is modelled by

$$y = f(x) = -0.001x^3 + 0.05x^2 - 0.4x + 12, \qquad 0 \le x \le 40,$$

and the far bank by the straight line $y = 0.15x + 28$. All distances are in metres.



(a) Find $f'(x)$. **[2]**

(b) Find the equation of the tangent to the near bank at the point where $x = 12$. Give your answer in the form $y = mx + c$. **[2]**

(c) Find the minimum width of the park, measured perpendicular to the road, and the value of x at which it occurs. **[2]**

(d) A footbridge, perpendicular to the road, is to be built across the river where the river is narrowest. By first writing down an expression for the width of the river, $W(x)$, find the value of x at which the footbridge should be built and the length of the footbridge. **[3]**

(e) Use the trapezoidal rule with 5 strips of equal width to estimate the area of the park. **[3]**

(f)

(i) Use your GDC to find the area of the park. **[1]**

(ii) Find the percentage error in your estimate from part (e). **[2]**

(g) Suggest how the estimate in part (e) could be made more accurate, still using the trapezoidal rule. **[1]**

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6. The masses of the eggs laid by the hens on a farm are normally distributed with mean 63 grams and standard deviation σ grams. An egg with a mass greater than 70 grams is classed as *large*. It is known that 15 % of the eggs are large.

(a) Show that $\sigma = 6.75$, correct to three significant figures. **[3]**

(b) An egg with a mass less than 53 grams is classed as *small*. Find the probability that an egg chosen at random is small. **[2]**

(c) The farm sells the heaviest 5 % of its eggs with a premium label. Find the minimum mass of an egg with a premium label. **[2]**

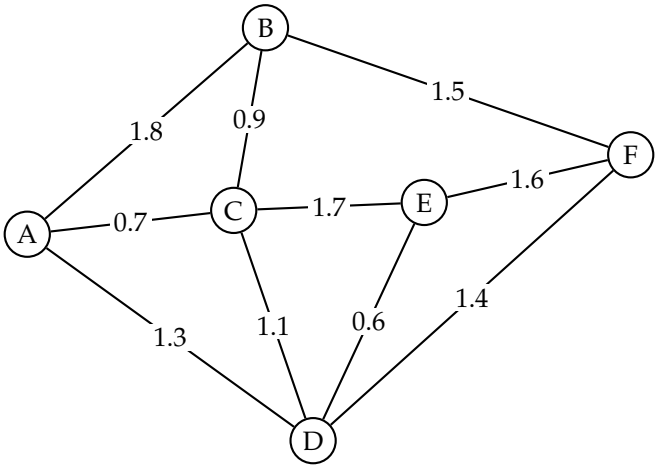
(d) The eggs are packed at random in boxes of 12. Find the probability that a box contains at most one large egg. **[2]**

(e) An egg is chosen at random. Given that its mass is greater than 60 grams, find the probability that it is large. **[3]**

(f) The farm can grade its eggs and sell them at 0.14 EUR for a small egg, 0.34 EUR for a large egg and 0.26 EUR for any other egg. Grading costs the farm 0.02 EUR per egg. Alternatively, a wholesaler offers to buy all the eggs ungraded for 0.25 EUR each.

By finding the expected income per egg from grading, after the cost of grading, determine which option the farm should choose. **[4]**

7. A nature reserve has a visitor centre, A, and five birdwatching hides, B, C, D, E and F, linked by trails. In the graph below the vertices represent the visitor centre and the hides, the edges represent the trails and the weights are the lengths of the trails, in kilometres.



(a) A cable for webcams is to be laid along some of the trails so that the visitor centre and all the hides are connected. Use Kruskal's algorithm to find the minimum length of cable needed. List the edges in the order in which you select them. **[3]**

(b) A ranger must walk along every trail at least once to inspect it, starting and finishing at the visitor centre.

(i) Write down the vertices of odd degree. **[1]**

(ii) By considering all possible pairings of these vertices, determine which trails the ranger should walk along twice. **[3]**

(iii) Find the length of the ranger's shortest route. **[1]**

A birdwatcher wants to start at the visitor centre, visit every hide and return to the visitor centre, travelling the shortest possible distance. The table of least distances, in kilometres, is shown below.

	A	B	C	D	E	F
A	–	x	0.7	1.3	1.9	y
B	x	–	0.9	2.0	2.6	1.5
C	0.7	0.9	–	1.1	1.7	2.4
D	1.3	2.0	1.1	–	0.6	1.4
E	1.9	2.6	1.7	0.6	–	1.6
F	y	1.5	2.4	1.4	1.6	–

(c) Write down the value of x and the value of y. **[2]**

(d) Use the nearest neighbour algorithm, starting at A, to find an upper bound for the length of the birdwatcher's route. State the order in which the vertices are visited. **[2]**

(e) By deleting vertex A, use the deleted vertex algorithm to find a lower bound for the length of the birdwatcher's route. **[3]**

Predicted paper · May 2027

IB Mathematics: applications and interpretation

Higher level · Paper 3

1 hour

Instructions

- 2 extended-response problem-solving questions, 55 marks. Answer on lined paper.
- Total: 55 marks. Answer all questions.
- A graphic display calculator is required for this paper.
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Mark scheme and the other papers in the set: scan the code

1. This question investigates how the bicycles of a town's bike-sharing scheme move between two hubs.

The scheme has 250 bicycles and two docking hubs, Old Town (O) and University (U). Every bicycle is back in a dock at one of the two hubs at the end of each day. The operator's records show that, of the bicycles at O at the start of a day, 18% are at U at the end of that day and the rest are at O. Of the bicycles at U at the start of a day, 27% are at O at the end of that day and the rest are at U.

At the start of day 1 there are 200 bicycles at O and 50 at U. Let $\mathbf{s}_n = \begin{pmatrix} o_n \\ u_n \end{pmatrix}$, where o_n and u_n are the expected numbers of bicycles at O and at U at the end of day n, so that $\mathbf{s}_0 = \begin{pmatrix} 200 \\ 50 \end{pmatrix}$ and $\mathbf{s}_n = \mathbf{T}^n \mathbf{s}_0$.

(a) Write down the transition matrix $\mathbf{T}$, in which each column gives the probabilities for a bicycle that starts the day at one hub. **[1]**

(b) Find the expected number of bicycles at U at the end of day 3. **[2]**

(c) Show that the eigenvalues of $\mathbf{T}$ are 1 and 0.55. **[2]**

(d) Find an eigenvector of $\mathbf{T}$ corresponding to each eigenvalue. **[2]**

(e) Hence show that $o_n = 150 + 50\,(0.55)^n$. **[3]**

(f)

(i) Write down the expected number of bicycles at each hub in the long term. **[1]**

(ii) Find the first day at the end of which the expected number of bicycles at O is less than 151. **[2]**

During the morning rush hour, riders take bicycles from one hub to the other and a van moves bicycles to the hub that needs them. The operator models the numbers of bicycles above the target levels at the two hubs. Let x be the number of bicycles above the target level at O and y the number above the target level at U, t hours after 07:00. A negative value means the hub is below its target level. The model is

$$\frac{\mathrm{d}x}{\mathrm{d}t} = -x + 0.4y, \qquad \frac{\mathrm{d}y}{\mathrm{d}t} = 0.3x - 0.6y,$$

with $x = 24$ and $y = -28$ at 07:00.

(g) Find the eigenvalues of the matrix $\begin{pmatrix} -1 & 0.4 \\ 0.3 & -0.6 \end{pmatrix}$ and a corresponding eigenvector for each eigenvalue. **[3]**

(h) State, with a reason, the type of equilibrium point at $(0,\ 0)$ and whether it is stable or unstable. **[2]**

(i) Find expressions for x and y in terms of t. **[3]**

(j)

(i) Use Euler's method with a step length of 0.25 hours to estimate the values of x and y at 08:00. **[3]**

(ii) Find the percentage error in your estimate of x at 08:00. **[1]**

(k) According to the model, the number of bicycles at O falls below its target level at about 08:44 and then stays below it. Find the least value of x predicted by the model and hence comment on whether the operator needs to plan an extra van trip to O later that morning. **[2]**

2. This question investigates whether new dispatch software has improved the response times of an ambulance service.

Before the software was introduced, the mean response time of the service (from the emergency call to the arrival of the ambulance) was 9.6 minutes, and the mean distance from the ambulance base to the incident was 6.1 km. After the introduction, the manager selects 10 calls at random from one week in July and records the response time, t minutes, and the distance, d km.

d (km)	4.1	6.8	5.2	9.5	3.4	2.4	7.1	3.8	4.6	5.9
t (min)	8.4	9.4	7.2	11.3	7.9	6.8	10.2	8.7	6.5	9.1

(a)

(i) Write down the mean response time of the sample. **[1]**

(ii) Show that an unbiased estimate of the variance of the response times is 2.32 minutes^2, correct to three significant figures. **[2]**

(b)

(i) State one assumption about the response times that is needed to construct a confidence interval for the mean using the t-distribution. **[1]**

(ii) Find a 95% confidence interval for the mean response time after the introduction of the software. **[2]**

(iii) The manager claims that the mean response time is now less than 9 minutes. Comment on this claim, using your answer to part (b)(ii). **[1]**

(c) The manager tests, at the 5% significance level, whether the mean response time has decreased.

(i) Write down the null and alternative hypotheses. **[1]**

(ii) Find the p-value for this test. **[2]**

(iii) State the conclusion of the test in context, giving a reason. **[1]**

(d) An arrival is classed as late if the response time is more than 15 minutes. Before the software, late arrivals occurred at a mean rate of 4.5 per week. The number of late arrivals in the first 4 weeks after the introduction, X, is used to test H_0: the mean rate is 4.5 per week, against H_1: the mean rate is less than 4.5 per week, at the 5% significance level.

(i) State one condition, in context, for X to be modelled by a Poisson distribution. **[1]**

(ii) Find the critical region for this test. **[3]**

(iii) Write down the probability of a type I error. **[1]**

(e) In the first 4 weeks there were 12 late arrivals. State the conclusion of the test, giving a reason. **[1]**

(f)

(i) Describe, in context, what a type II error would be in this test. **[1]**

(ii) Given that the mean rate of late arrivals after the introduction is in fact 2.5 per week, find the probability of a type II error. **[2]**

(iii) State how the probability of a type II error could be reduced without changing the significance level. **[1]**

(g)

(i) Find Spearman's rank correlation coefficient, r_s, between d and t for the 10 calls. **[2]**

(ii) Interpret your value of r_s in context. **[1]**

(iii) Suggest a reason why r_s may be more appropriate than Pearson's correlation coefficient for these data. **[1]**

(h) The manager concludes that the software has reduced the mean response time. Using your answers to parts (b), (c) and (g), and the information about the sample, evaluate this conclusion. **[3]**

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