Topic test · IB Mathematics: analysis and approaches

Calculus

Standard level

40 minutes

Instructions

- 6 questions, 35 marks, on SL 5.1 to 5.11 (HL: also AHL 5.12 to 5.17). Answer in the spaces provided.
- Total: 35 marks. Answer all questions.
- A graphic display calculator is allowed. You need a clean copy of the formula booklet.
- Show your working: most marks go to the method, not to the final number.
- Sit it in one go, timed. Then mark it with the mark scheme, honestly.

Name .....

Date .....

Score .....

Every question in this test is new, written by Pietro Meloni in the style of the current papers, to check one topic of the syllabus under timed conditions. It is not IB material and not endorsed by the IB.

Mark scheme and the other tests: scan the code

1. Consider the function $f(x) = x^3 - 4x^2 + 6$.

(a) Find $f'(x)$. **[1]**

(b) Find the equation of the tangent to the graph of f at the point where $x = 3$. Give your answer in the form $y = mx + c$. **[3]**

2. Consider the function $g(x) = \ln\left(x^2 - 3\right)$, for $x > \sqrt{3}$. The point $\mathrm{P}(2, 0)$ lies on the graph of g.

(a) Find $g'(x)$. **[2]**

(b) Find the equation of the normal to the graph of g at P. Give your answer in the form $y = mx + c$. **[3]**

3. Consider the function $h(x) = x^4 - 4x^3$, for $x \in \mathbb{R}$.

(a) Find the coordinates of the two stationary points of the graph of h. **[3]**

(b) Determine the nature of each stationary point. Justify your answers. **[3]**

4. The gradient of a curve $y = f(x)$ is given by $f'(x) = 3x^2 - 6x + 2$. The curve passes through the point $(3, 6)$.

(a) Find $f(x)$. **[4]**

(b) The curve and the x-axis enclose two regions. Find the exact value of the total area of the two regions. **[3]**

5. A gardener makes an open-top planter in the shape of a cuboid from 5400 cm^2 of thin sheet metal. The base of the planter is a rectangle measuring x cm by $2x$ cm, and the height of the planter is h cm. The planter has no lid and all of the metal is used.

(a) Show that the volume of the planter, V cm^3, is given by $V = 1800x - \frac{2}{3}x^3$. **[3]**

(b) Find the value of x for which V is a maximum. **[3]**

(c) Justify that this value of x gives a maximum, and find the maximum volume. **[2]**

6. A particle moves along a straight line. Its velocity, v m s^{-1}, at time t seconds is given by $v(t) = t^2 - 6t + 8$, for $0 \leq t \leq 5$.

(a) Find the displacement of the particle from its initial position at $t = 5$. **[2]**

(b) Find the total distance travelled by the particle in the first 5 seconds. **[3]**