Predicted paper · May 2027

IB Mathematics: analysis and approaches

Higher level · Paper 1

2 hours

Instructions

- Section A: 9 short questions, 55 marks. Answer in the spaces provided.
- Section B: 3 longer questions, 55 marks. Answer on lined paper.
- Total: 110 marks. Answer all questions.
- No calculator is allowed for this paper.
- Full marks are not necessarily awarded for a correct answer with no working: show your method.
- You need a clean copy of the mathematics: analysis and approaches formula booklet.
- Sit it in one go, timed. Then mark it with the mark scheme, honestly.

Name .....

Date .....

Score .....

Every question in this paper is new, written by Pietro Meloni to the shape of the papers set on the current syllabus. It is not IB material and it is not a leaked paper: it is a prediction of the topics and the style, for practice under timed conditions.

Mark scheme and the other papers in the set: scan the code

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Section A

1. The function f is defined by $f(x) = x^2 - 6x + 5$, $x \in \mathbb{R}$.

(a) Find the zeros of f. **[2]**

(b) Express $f(x)$ in the form $(x-h)^2 + k$, where $h, k \in \mathbb{Z}$. **[2]**

(c) Write down the coordinates of the vertex of the graph of $y = f(x)$. **[1]**

2. The third term of an arithmetic sequence $u_1, u_2, u_3, \ldots$ is 11 and the eighth term is 31.

(a) Find the common difference and the first term of the sequence. **[3]**

(b) Find the value of n for which $\sum_{k=1}^{n} u_k = 465$. **[3]**

3. In a group of 30 students, 18 study Spanish, 14 study German and 5 study neither of these two languages. A student is chosen at random from the group. Let S be the event that the student studies Spanish and G the event that the student studies German.

(a) Find the number of students who study both languages. **[2]**

(b) Find $\mathrm{P}(G \mid S)$. **[2]**

(c) Determine, with a reason, whether the events S and G are independent. **[2]**

4. Let $f(x) = \cos 2x + 3\sin x$.

(a) Find the exact value of $f\left(\frac{\pi}{6}\right)$. **[2]**

(b) Show that $f(x) = 1 + 3\sin x - 2\sin^2 x$. **[2]**

(c) Hence solve the equation $f(x) = 2$ for $0 \leq x \leq 2\pi$. **[3]**

5. Let $f(x) = x \ln x$, where $x > 0$.

(a) Find $f'(x)$. **[2]**

(b) Find the equation of the normal to the graph of f at the point where $x = \mathrm{e}$. Give your answer in the form $y = mx + c$. **[4]**

(c) Find the exact coordinates of the stationary point of the graph of f. **[2]**

6. Consider the expansion of $\left(2x - \frac{1}{x^2}\right)^6$.

(a) Find the term that is independent of x. **[4]**

(b) Hence, or otherwise, find the coefficient of x^3 in the expansion of $(1 + x^3)\left(2x - \frac{1}{x^2}\right)^6$. **[4]**

7. Let $z = 1 - \sqrt{3}\,\mathrm{i}$ and let z^* denote the complex conjugate of z.

(a) Find $|z|$ and $\arg z$. **[2]**

(b) Find $\dfrac{z}{z^*}$, giving your answer in the form $re^{\mathrm{i}\theta}$, where $r > 0$ and $-\pi < \theta \leq \pi$. **[3]**

8. The polynomial $p(x) = x^3 + ax^2 + bx - 12$, where $a, b \in \mathbb{Z}$, has a factor $(x - 2)$ and leaves a remainder of -6 when divided by $(x + 1)$.

(a) Find the value of a and the value of b. **[3]**

(b) Hence find all the solutions of the equation $p(x) = 0$. **[2]**

9. Consider the vectors $\mathbf{a} = \begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}$.

(**a**) Find the angle between $\mathbf{a}$ and $\mathbf{b}$. **[3]**

(**b**) Find the value of t for which the vector $\mathbf{a} + t\mathbf{b}$ is perpendicular to $\mathbf{a}$. **[2]**

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Section B

1. Let $f(x) = \dfrac{2x+3}{x^2+4}$, $x \in \mathbb{R}$.

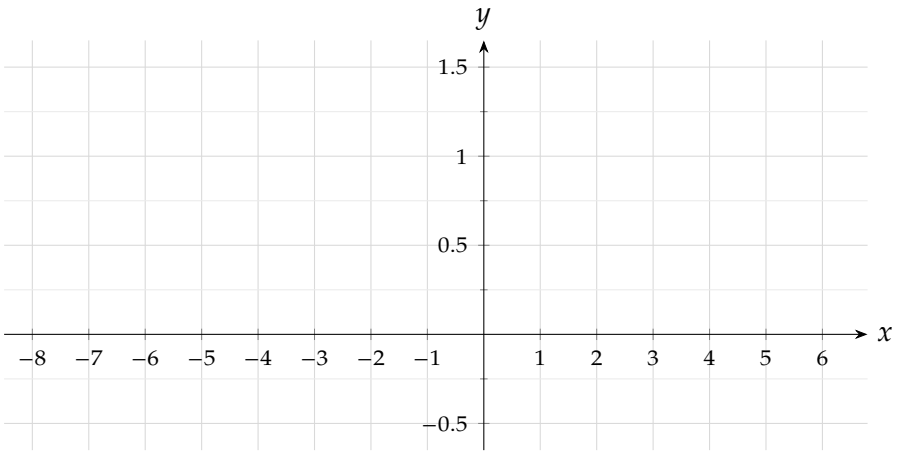
(a) Find the coordinates of the points where the graph of $y = f(x)$ crosses the axes. **[2]**

(b) Write down the equation of the horizontal asymptote of the graph of $y = f(x)$. **[1]**

(c) Show that $f'(x) = \dfrac{-2(x+4)(x-1)}{(x^2+4)^2}$. **[3]**

(d) Find the coordinates of the stationary points of the graph of $y = f(x)$ and determine their nature. **[4]**

(e) On the axes below, sketch the graph of $y = f(x)$ for $-8 \leq x \leq 6$, showing clearly the stationary points and the points where the graph crosses the axes. **[3]**



(f) Find the exact area of the region enclosed by the graph of $y = f(x)$, the x-axis and the lines $x = 0$ and $x = 2$. **[4]**

2. The sequence $u_1, u_2, u_3, \ldots$ is defined by

$$u_1 = 2, \qquad u_{n+1} = 2 - \frac{1}{u_n} \quad \text{for } n \in \mathbb{Z}^+.$$

(a) Find u_2, u_3 and u_4. **[2]**

(b) Hence suggest an expression for u_n in terms of n. **[1]**

(c) Prove your conjecture from part (b) by mathematical induction. **[6]**

(d) Show that $u_n - u_{n+1} = \dfrac{1}{n(n+1)}$. **[2]**

(e) Hence show that $\displaystyle\sum_{r=1}^{n} \frac{1}{r(r+1)} = \frac{n}{n+1}$. **[2]**

(f) Consider now the series $\displaystyle\sum_{r=1}^{\infty} \frac{1}{r(r+2)}$.

(i) Express $\frac{1}{r(r+2)}$ in partial fractions. **[2]**

(ii) Hence find the exact value of $\sum_{r=1}^{\infty} \frac{1}{r(r+2)}$. **[3]**

3. The points A$(1, 0, -1)$, B$(0, 1, 3)$ and C$(2, 1, -1)$ lie on the plane Π_1.

(a) Find $\overrightarrow{AB} \times \overrightarrow{AC}$. **[3]**

(b) Hence show that the Cartesian equation of Π_1 is $2x - 2y + z = 1$. **[2]**

The line L has equation $\mathbf{r} = \begin{pmatrix} 0 \\ 2 \\ -1 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}, \lambda \in \mathbb{R}$.

(c) Find the coordinates of the point P where L meets Π_1. **[3]**

(d) Find the acute angle between L and Π_1. **[3]**

The plane Π_2 has equation $x + 2y + 2z = 5$.

(e) Show that Π_1 and Π_2 are perpendicular. **[2]**

(f) The planes Π_1 and Π_2 intersect in the line M. Find a vector equation of M. **[3]**

The perpendicular distance from the point (x_0, y_0, z_0) to the plane $ax + by + cz = d$ is

$$\frac{|ax_0 + by_0 + cz_0 - d|}{\sqrt{a^2 + b^2 + c^2}}.$$

(g) The point Q lies on L and is equidistant from Π_1 and Π_2. Find the coordinates of Q. **[4]**

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Predicted paper · May 2027

IB Mathematics: analysis and approaches

Higher level · Paper 2

2 hours

Instructions

- Section A: 9 short questions, 55 marks. Answer in the spaces provided.
- Section B: 3 longer questions, 55 marks. Answer on lined paper.
- Total: 110 marks. Answer all questions.
- A graphic display calculator is required for this paper.
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Mark scheme and the other papers in the set: scan the code

Section A

1. Lena invests 8000 USD in a savings account that pays a nominal annual interest rate of 3.6%, compounded monthly. No further deposits or withdrawals are made.

(**a**) Find the value of the investment after 5 years. Give your answer correct to the nearest cent. **[2]**

(**b**) Find the minimum number of complete years needed for the value of the investment to exceed 12 000 USD. **[3]**

2. A triangular plot of land is shown in the diagram, where AB = 85 m, BC = 62 m and $A\hat{B}C = 58°$.

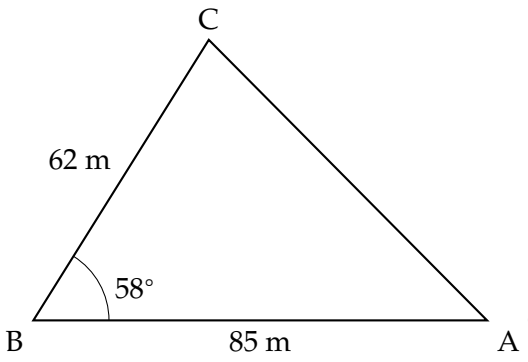


diagram not to scale

(**a**) Find the length of AC. **[2]**

(**b**) Find the size of $B\hat{A}C$. **[2]**

(**c**) The land is valued at 12 USD per square metre. Find the value of the plot. **[2]**

3. The masses of the avocados grown in an orchard are normally distributed with mean 185 g and standard deviation 22 g.

(**a**) Find the probability that a randomly chosen avocado has a mass greater than 200 g. **[2]**

(**b**) The heaviest 15% of the avocados are sold as *premium*. Find the minimum mass of a premium avocado. **[2]**

(**c**) The masses of the avocados grown in a second orchard are also normally distributed, with mean 190 g and standard deviation σ g. In this orchard, 10% of the avocados have a mass less than 160 g. Find the value of σ. **[3]**

4. A cup of coffee is left to cool in a room. Its temperature T, in degrees Celsius, after t minutes is modelled by

$$T(t) = 21 + A\,\mathrm{e}^{-kt}, \qquad t \geq 0,$$

where A and k are positive constants. The initial temperature of the coffee is 85°C and after 10 minutes its temperature is 60°C.

(**a**) Write down the value of A. **[1]**

(b) Find the value of k. **[2]**

(c) Find the time taken for the temperature of the coffee to fall to 30°C. **[2]**

(d) Write down the equation of the horizontal asymptote of the graph of T, and state what it represents in this context. **[2]**

5. A closed cylindrical can has radius r cm and height h cm. The volume of the can is 330 cm^3.

(a) Show that the total surface area, S cm^2, of the can is given by $S = 2\pi r^2 + \frac{660}{r}$. **[2]**

(b) Find $\frac{\mathrm{d}S}{\mathrm{d}r}$. **[2]**

(c) Find the value of r for which S is a minimum, and the corresponding minimum surface area. **[3]**

6. A factory produces rechargeable batteries. Each battery, independently of the others, fails a quality test with probability 0.06. The batteries are packed in boxes of 20. Let X be the number of batteries in a randomly chosen box that fail the test.

(a) Find the probability that, in a randomly chosen box,

(i) exactly two batteries fail the test; **[2]**

(ii) at least three batteries fail the test. **[2]**

(b) Find the expected number of batteries in a box that fail the test. **[1]**

(c) An inspector selects n batteries at random from the production line. Find the smallest value of n such that the probability that at least two of these batteries fail the test is greater than 0.9. **[3]**

7. A water tank has the shape of an inverted right circular cone with its vertex at the bottom. The radius of the top of the tank is 1.2 m and its height is 3 m. The depth of the water in the tank is h metres and the volume of the water is V cubic metres.

(a) Show that $V = \frac{4\pi}{75}h^3$. **[2]**

(b) Water leaks out of the tank at a constant rate of 0.05 m^3 per minute. Find the rate at which the

depth of the water is decreasing at the instant when the depth is 2 m. **[3]**

8. A workshop makes a component using three 3D printers, P, Q and R. Printer P makes 50% of the components, printer Q makes 30% and printer R makes 20%. The probabilities that a component is defective are 0.01 for printer P, 0.02 for printer Q and 0.04 for printer R.

A component is chosen at random from the output of the workshop.

(a) Find the probability that the component is defective. **[2]**

(b) Given that the component is defective, find the probability that it was made by printer R. **[3]**

9. The lines L_1 and L_2 have equations

$$L_1 : \mathbf{r} = \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix}, \qquad L_2 : \mathbf{r} = \begin{pmatrix} 1 \\ -1 \\ 6 \end{pmatrix} + \mu \begin{pmatrix} 1 \\ 1 \\ -2 \end{pmatrix}, \qquad \lambda, \mu \in \mathbb{R}.$$

The point A has coordinates $(1, 2, -1)$ and the point B has coordinates $(1, -1, 6)$.

(a) Show that L_1 and L_2 intersect, and find the coordinates of their point of intersection, P. **[3]**

(b) Find the area of triangle ABP. **[2]**

Section B

1. The time, X hours, that a visitor spends at an indoor climbing gym is modelled by a continuous random variable with probability density function

$$f(x) = \begin{cases} k\,x^2(4-x), & 0 \le x \le 4, \\ 0, & \text{otherwise,} \end{cases}$$

where k is a positive constant.

(a) Show that $k = \dfrac{3}{64}$. **[2]**

(b) Find the probability that a visitor spends between 1 and 3 hours at the gym. **[2]**

(c) Find

(i) $\mathrm{E}(X)$; **[2]**

(ii) $\mathrm{Var}(X)$. **[3]**

(d) Find the median of X. **[3]**

(e) The gym charges a flat fee of 8 dollars plus 5 dollars for each hour spent at the gym, so that a visitor pays $C = 8 + 5X$ dollars. Find the expected value and the standard deviation of C. **[2]**

(f) On a particular morning 10 visitors arrive at the gym. The times they spend at the gym are independent of each other. Find the probability that at least 3 of these visitors spend more than 3 hours at the gym. **[3]**

2. A tank on a vineyard is used to prepare liquid fertiliser. Initially the tank contains 400 litres of water in which 2 kg of fertiliser is dissolved. From time $t = 0$, a solution containing 0.02 kg of fertiliser per litre flows into the tank at a constant rate of 6 litres per minute. The contents are kept well mixed and the mixture flows out of the tank at a constant rate of 4 litres per minute.

Let M be the mass of fertiliser, in kg, in the tank t minutes after the flow starts.

(a) Show that $\dfrac{\mathrm{d}M}{\mathrm{d}t} = 0.12 - \dfrac{2M}{200 + t}$. **[3]**

(b) Use Euler's method with a step length of 10 minutes to estimate the mass of fertiliser in the tank after 30 minutes. **[3]**

(c)

(i) Show that $(200+t)^2$ is an integrating factor for the differential equation in part (a). **[2]**

(ii) Hence find M in terms of t. **[4]**

(d) Find the percentage error in the estimate found in part (b). **[2]**

(e) The tank can hold 1000 litres. Find the concentration of fertiliser, in kg per litre, in the tank at the moment it becomes full. **[2]**

(f) Show that, however large the tank, the concentration of fertiliser in the tank never reaches 0.02 kg per litre. **[2]**

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3. The points $A(1,-1,3)$, $B(3,1,2)$ and $C(2,3,4)$ lie on the plane Π.

(a) Find $\overrightarrow{AB} \times \overrightarrow{AC}$. **[3]**

(b) Hence show that a Cartesian equation of Π is $2x - y + 2z = 9$. **[2]**

The line L passes through the points $D(1,2,0)$ and $F(3,8,4)$.

(c) Find a vector equation of L. **[2]**

(d) The line L meets the plane Π at the point E. Find the coordinates of E. **[3]**

(e) Find the acute angle between L and Π. **[3]**

(f) Find the distance from D to Π. **[2]**

(g) The point D' is the reflection of D in the plane Π. Find the coordinates of D'. **[3]**

(h) Hence find a vector equation of the line which is the reflection of L in the plane Π. **[2]**

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Predicted paper · May 2027

IB Mathematics: analysis and approaches

Higher level · Paper 3

1 hour

Instructions

- 2 extended-response problem-solving questions, 55 marks. Answer on lined paper.
- Total: 55 marks. Answer all questions.
- A graphic display calculator is required for this paper.
- Full marks are not necessarily awarded for a correct answer with no working: show your method.
- You need a clean copy of the mathematics: analysis and approaches formula booklet.

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1. This question investigates the family of functions $f_n(x) = x^n e^{-x}$, $x \geq 0$, where $n \in \mathbb{Z}^+$, and uses a related family of integrals to obtain a result about the number e.

(a) Use your GDC to find the coordinates of the local maximum point on the graph of $y = f_3(x)$. **[2]**

(b) Show that $f_n'(x) = x^{n-1}e^{-x}(n - x)$. **[2]**

(c) Hence find, in terms of n, the coordinates of the local maximum point on the graph of $y = f_n(x)$. **[2]**

(d)

(i) Show that $f_n''(x) = x^{n-2}e^{-x}(x^2 - 2nx + n(n-1))$. **[2]**

(ii) Hence show that, for $n \geq 2$, the graph of $y = f_n(x)$ has exactly two points of inflexion with $x > 0$, at $x = n \pm \sqrt{n}$. Justify your answer. **[3]**

Let $I_n = \int_0^1 x^n e^{-x}\, dx$, for $n \in \mathbb{N}$.

(e)

(i) Show that $I_0 = 1 - e^{-1}$. **[1]**

(ii) Show that $I_n = nI_{n-1} - e^{-1}$ for $n \geq 1$. **[3]**

(f) Use the result in part (e) to find the exact value of I_3. Verify your answer using your GDC. **[3]**

(g) Prove by mathematical induction that $I_n = n!\left(1 - e^{-1}\sum_{k=0}^{n}\frac{1}{k!}\right)$ for all $n \in \mathbb{N}$. **[5]**

(h)

(i) By considering the values of e^{-x} for $0 \leq x \leq 1$, show that $0 \leq I_n \leq \frac{1}{n+1}$. **[2]**

(ii) Hence show that $\lim_{n\to\infty}\sum_{k=0}^{n}\frac{1}{k!} = e$. **[2]**

2. This question investigates the distances between the vertices of a regular polygon drawn on the Argand diagram.

Let $n \geq 3$ be an integer and let $\omega = e^{\frac{2\pi i}{n}}$. The nth roots of unity $1, \omega, \omega^2, \ldots, \omega^{n-1}$ are the vertices of a regular polygon inscribed in the circle $|z| = 1$. Let A be the vertex $z = 1$ and, for $k = 1, 2, \ldots, n-1$, let $d_k = |1 - \omega^k|$ be the distance from A to the vertex ω^k.

(a) For $n = 5$, write down the five roots of $z^5 = 1$ in the form $e^{i\theta}$, where $0 \leq \theta < 2\pi$. **[1]**

(b) Show that, for any $n \geq 3$, the sum of the nth roots of unity is zero. **[2]**

(c) For $n = 5$, use your GDC to find the value of $d_1 d_2 d_3 d_4$. **[2]**

(d) Consider the case $n = 6$.

(i) Find the exact values of d_1, d_2, d_3, d_4, d_5, and hence find the value of $d_1 d_2 d_3 d_4 d_5$. **[2]**

(ii) Using your answers to parts (c) and (d)(i), make a conjecture about the value of $d_1 d_2 \cdots d_{n-1}$. **[1]**

(e) Show that $\left|1 - e^{i\theta}\right| = 2\sin\frac{\theta}{2}$ for $0 < \theta < 2\pi$. **[3]**

(f) By writing $z^n - 1$ as a product of n linear factors and also as $(z-1)\left(1 + z + z^2 + \cdots + z^{n-1}\right)$, prove your conjecture from part (d)(ii). **[4]**

(g) Hence show that $\prod_{k=1}^{n-1} \sin\frac{k\pi}{n} = \frac{n}{2^{n-1}}$. **[2]**

Let $S_n = d_1 + d_2 + \cdots + d_{n-1}$ be the sum of the distances from A to the other $n-1$ vertices.

(h)

(i) Show that $\sum_{k=0}^{n-1} e^{ik\pi/n} = \frac{2}{1 - e^{i\pi/n}}$. **[2]**

(ii) Show that $\frac{2}{1 - e^{i\varphi}} = 1 + i\cot\frac{\varphi}{2}$ for $0 < \varphi < 2\pi$. **[3]**

(iii) Hence show that $S_n = 2\cot\frac{\pi}{2n}$. **[2]**

(i) Let $m_n = \frac{S_n}{n-1}$ be the mean distance from A to the other vertices.

(i) Find m_{12}. **[1]**

(ii) Find the exact value of $\lim_{n\to\infty} m_n$. **[3]**